1. Determine whether each of the following integrals is convergent (C) or divergent (D). Evaluate those which are convergent.

(a) The anti-derivative of \(x^{-2/3}\) is \(3x^{1/3}\). Hence
\[
\int_{-1}^{1} x^{-2/3} \, dx = \left( \lim_{t \to 0^-} 3x^{1/3}|_{-1}^{t} \right) + \left( \lim_{t \to 0^+} 3x^{1/3}|_{t}^{1} \right) = (0 + 3) + (3 - 0) = 6,
\]
and the integral converges.

(b) The anti-derivative of \(1/\sqrt{x} = x^{-3/2}\) is \(-2x^{-1/2}\). Hence
\[
\int_{0}^{3} \frac{1}{\sqrt{x}} \, dx = \lim_{t \to 0^+} -2x^{-1/2}|_{t}^{3} = \lim_{t \to 0^+} (-2(3)^{-1/2} + 2t^{-1/2}) \to \infty.
\]
The integral diverges.

(c) The anti-derivative of \(xe^{-x^2}\) is \(-\frac{1}{2}e^{-x^2}\), and
\[
\int_{0}^{\infty} xe^{-x^2} \, dx = \lim_{t \to \infty} \frac{1}{2}e^{-x^2}|_{0}^{t} = \lim_{t \to \infty} \left(-\frac{1}{2}e^{-t^2} + \frac{1}{2}\right) = \frac{1}{2}.
\]
The integral converges.

2. (a) Set-up the integral which computes the arc length of the curve \(y = 2x^{3/2}\), \(0 \leq x \leq 1\).
\[
\int_{0}^{1} \sqrt{1 + 9x} \, dx = \frac{2}{27} \left(\sqrt{1000} - 1\right).
\]
(b) Evaluate the above integral. See above.

3. (a) Consider the curve given by the parametric equation
\[
x = \cos(t) + t \sin(t) \quad y = \sin(t) - t \cos(t) \quad 1 \leq t \leq 3.
\]
Find the length of this curve.
\[
\int_{1}^{3} t \, dt = 4.
\]
(b) Consider the surface obtained by rotating the above curve about the y-axis. Set-up
the integral which computes the surface area.
\[
SA = 2\pi \int_1^3 x \, ds = 2\pi \int_1^3 t \cos(t) + t^2 \sin(t) \, dt.
\]

4. (20 pts) (a) Determine the slope, i.e. \( \frac{dy}{dx} \), at time \( t \) for the parametric curve
\[
x = \sin(t + \sin(t)) \quad y = \cos(t + \cos(t)).
\]
\[
\frac{dy}{dx} = \frac{y'}{x'} = \frac{-\sin(t + \cos(t))(1 - \sin(t))}{\cos(t + \sin(t))(1 + \cos(t))}
\]

(b) Find the equation of the tangent line to the above curve at time \( t = 0 \).
At \( t = 0 \) the point is \((x, y) = (0, \cos(1))\), and the slope is \( m = \frac{-
\sin(1)}{2} \). Whence the equation of the line is
\[
y = \frac{-\sin(1)}{2} x + \cos(1).
\]

5. (a) Sketch the curve given by the polar equation
\[
r = 4 - \sin(\theta).
\]

This is a large egg-shaped region intersecting the coordinate axes at the four points
\((0, \pm 4), (0, 3)\) and \((0, -5)\).

(b) Find the area of the region inside of the above curve.
\[
A = \frac{1}{2} \int_0^{2\pi} r^2 \, d\theta = \frac{1}{2} \int_0^{2\pi} (16 - 8 \sin \theta + \sin^2 \theta) \, d\theta = \frac{33\pi}{2}
\]

6. Determine whether each of the following sequences \( \{a_n\}_{n=1}^\infty \) is convergent (C) or
divergent (D). Find the limit of those which are convergent.

(a) \( a_n = \frac{\sqrt{n}}{2 + \sqrt{n}} \).
Convergent: \( a_n \to 1 \).

(b) \( a_n = \sqrt{n} - \sqrt{n^2 - 1} \).
Divergent: In fact \( a_n \to -\infty \).
(c) $a_n = \frac{3n^4 + 6n + 1}{5n^3 - 4n^2 + 2n}$.  
Convergent: $a_n \to 0$.

7. (a) The following series converges. Find its sum.

$$
\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} = \sum_{n=1}^{\infty} \frac{1}{2} \left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{\infty} \frac{1}{3} \left(\frac{1}{3}\right)^{n-1} = \frac{1/2}{1 - 1/2} + \frac{1/3}{1 - 1/3} = 1 + 1/2 = \frac{3}{2}.
$$

(b) Express the infinite repeating decimal $0.161616\ldots$ as a rational number (i.e. fraction) in the reduced form $\frac{p}{q}$ where $p$ and $q$ are positive integers and have no common factors.

$$
0.161616\ldots = 0.16 + 0.0016 + 0.000016 + 0.00000016 + \cdots = \frac{16}{100} + \frac{16}{10000} + \frac{16}{1000000} + \cdots = \frac{16}{100} + \frac{16}{100^2} + \frac{16}{100^3} + \frac{16}{100^4} + \cdots = \sum_{n=1}^{\infty} 16 \left(\frac{1}{100}\right)^n = \sum_{n=1}^{\infty} \frac{16}{100} \left(\frac{1}{100}\right)^{n-1} = \frac{16/100}{1 - 1/100} = \frac{16}{99}.
$$

8. (25 pts) Determine whether each of the following series is convergent (C) or divergent (D) and briefly explain why.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n}$.

Convergent. Geometric Series with $r = -1/3$.

(b) $\sum_{n=1}^{\infty} \frac{1}{n^3}$.

Convergent. $p$-Series with $p = 3$.

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

Divergent. $p$-Series with $p = 1/2$.

(d) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
Convergent. Integral Test or Telescoping Series.

\[ (e) \sum_{n=2}^{\infty} \frac{1}{n \ln(n)}. \]

Divergent. Integral Test.