NAME (please print legibly): ____________________________________________________
Your University ID Number: __________________________________________________
Circle your Instructor’s Name along with the Lecture Time:

Mike Gage (10 MWF)  Carl Mueller (9 MWF)

• Calculators are NOT allowed on this exam, but you do not have to simplify numerical answers; for example $\pi \sqrt{5}$ or even $34g\sqrt{5}$ joules ($g = 9.8\, m/sec^2$) are acceptable answers.

• Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.

• Please put your final answers in the spaces provided.

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<th>QUESTION</th>
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1. (10 points) Find the length of the curve

\[ y = \int_0^x \sqrt{t^2 - 1} \, dt \]

for \( x \) between 1 and 4.

ANSWER: ________________________________

2. (10 points)

(a) Specify all of the values of \( p \) such that

\[ \int_0^1 \frac{1}{x^p} \, dx < \infty \]

ANSWER: ________________________________

(b) Specify all of the values of \( p \) such that

\[ \int_1^\infty \frac{1}{x^p} \, dx < \infty \]

ANSWER: ________________________________
3. (15 points)

(a) Which of the following integrals is larger? To get full credit, you must give a reason.

\[ \int_0^1 \frac{1}{x^2} \, dx \quad \int_0^1 e^{x^2} \, dx \]

ANSWER: 

(b) Determine whether

\[ \int_0^1 \frac{e^{x^2}}{x^2} \, dx \]

is finite or infinite. To receive full credit, you must give a reason.

ANSWER: 


3
4. (10 points)

Find the surface area obtained by rotating the following curve about the y-axis.

\[ y = \frac{1}{3}(x^2 + 2)^{3/2} \quad 1 \leq x \leq 2 \]

ANSWER: 

\[ \]
5. (10 points)

Determine \( \frac{dy}{dx} \) for the parametric curve

\[
x = \cos t - \sin 3t \quad y = \sin t + \cos 2t.
\]

Express your answer as a function of \( t \).
6. (15 points)

(a) Sketch the region contained inside the two curves

\[ r = 2 \quad \text{and} \quad r = 4 \cos \theta. \]

(b) Set-up, but do not evaluate, an integral to compute the area of the region in part (a).

ANSWER: __________________________________________________________________________

6
7. (20 points) Match a parametric equation to each of the four graphs below by placing the number for the correct equation in the lettered blank.

A: ______________________ B: ______________________

C: ______________________ D: ______________________

1. $x = \cos(t)$  
   $y = \cos(2t)$  
   8. $x = t + \sin t$  
   $y = \cos t$

2. $x = \cos(t)$  
   $y = \sin(2t)$  
   9. $x = t + \sin t$  
   $y = \cos(t) + 1$

3. $x = \cos(t)$  
   $y = \sin(3t)$  
   10. $x = t + \sin t$  
    $y = \cos(t) + t$

4. $x = \cos(t)$  
   $y = \sin(4t)$  
   11. $x = 3 \cos(t) + 2$  
    $y = 2 \sin(t) - 1$

5. $x = t \cos(t)$  
   $y = t \sin(t)$  
   12. $x = 3 \cos(t) - 2$  
    $y = 2 \sin(t) + 1$

6. $x = e^{t/10} \cos(t)$  
   $y = e^{t/10} \sin(t)$  
   13. $x = 2 \cos(t) + 3$  
    $y = -\sin(t) + 2$

7. $x = e^{-t/10} \cos(t)$  
   $y = e^{-t/10} \sin(t)$