Goals:
(1) To get practice in computing work.
(2) To study work on a conceptual level and find a principle which simplifies computations.

Problem 1.
(a) Suppose that an object of mass $M$ is hanging 25 meters below the top of the cliff from a very light fishing line. How much work is done raising the mass to the top of the cliff?
(b) Suppose a cable of length 50 meters is hanging over the edge of a cliff. Assume that the rope has density 3 kg per meter. What is the work necessary to raise the rope to the top of the cliff? Does the answer depend on how tall the cliff is?
(c) What is the total mass $M$ of the rope?

Problem 2. Suppose that a rectangular tank of water has a base of dimensions 6 meters by 5 meters, and that the height of the tank is 4 meters. The tank is completely full of water. Remember that the density of water is 1000 kg/m$^3$.
(a) How much work would be done in raising an object of mass $M$ from the center of the tank to a level 2 meters higher than the top of the tank?
(b) Compute the work needed to pump the water up to a level 2 meters higher than the top of the tank.
(c) What is the total mass $M$ of the water?

Problem 3. Based on your evidence from problems 1 and 2, formulate a principle which would help in computing work. In the case of a tank, would your principle work for any kind of tank? What kind of a shape is required?

Problem 4. Try to justify your principle by doing a thought experiment. The answer is up to you, but here is something to consider. For the rope problem, suppose we cut the rope into small pieces, each of equal length. Consider two pieces, one at the level of the cliff and
one 50 meters below the level of the cliff. What is the total work done in bringing both pieces of rope to a level 25 meters below the top of the cliff? (This level is at the middle of the rope). Expand on this reasoning to justify your principle. How would this reasoning apply to the tank of water? Can you see now what restrictions are necessary on the shape of the tank?

Problem 5. This principle can also be used for nonsymmetric bodies. To illustrate the idea, consider the rope problem again.

(a) Write down the integral for the work done in raising the rope to the top of the cliff.

(b) Divide this integral by $Mg$, where $M$ is the total mass of the rope. Notice that $M$ can also be expressed as an integral.

(c) What would you have to multiply $Mg$ by to get the work necessary to raise the rope to the top of the cliff? This quantity is called the center of mass.

(d) Suppose the density $D(h)$ of the rope is not constant, but depends on the distance $h$ from the top of the cliff. How would you modify your formula for the center of mass to deal with this case?