Workshop 5  Math 162  Fall 2005

Goals:

(1) To review and organize the techniques used for finding anti-derivatives.
(2) (A review of section 7.5 in Stewart is relevant to this workshop.)

Problem 1. Some partial fraction problems:

\[
\int \frac{2x}{(x + 3)(3x + 1)} \, dx \quad \int \frac{2}{x^2 + 3x - 4} \, dx \\
\int \frac{x^3 - 4x - 10}{x^2 - x - 6} \, dx \quad \int \frac{x^2}{(x - 3)(x + 2)^2} \, dx
\]

What would be the hardest part of determining

\[
\int \frac{2x - 1}{x^3 - 3x + 1} \, dx
\]

(Given enough time how might you do it? Don’t actually solve for this explicitly.)

Problem 2.

Discuss the most promising strategies for each of these integrals with your group before you start to work out the problem. Is there more than one strategy that would work? Have a plan!

\[
\int \frac{\tan^2(x)}{\cos^2(x)} \, dx \quad \int \frac{\tan^2(x)}{\cos(x)} \, dx \\
\int x \sin^2(x) \, dx \quad \int e^{\sqrt{x}} \, dx \\
\int \frac{dx}{x^3 - 1}
\]
Problem 3. Is “undetermined coefficients” a legitimate way to solve this problem?

\[ \int (x^2 - x + 1) \ln(x) \, dx = (Ax^3 + Bx^2 + Cx + D) \ln x + Ex^3 + Fx^2 + Gx + H \]

How do you determine the coefficients? What happens if you guess wrong? How does this relate to the partial fractions decomposition technique?

What is another approach for finding the anti-derivative? Discuss with your group which method is better – or more accurately – what are the advantages and disadvantages of each method. Is one of these the right method?

Problem 4. (See section on page 509)

What kind of function is the anti-derivative of a polynomial?

Can the anti-derivative of one over a polynomial always be written as a polynomial? Sometimes? What new functions do you need to use, besides polynomials to describe the derivative of 1 over a polynomial?

Are there any continuous functions that do not have simple-form anti-derivatives? (A ‘simple-form’ function is one that can be written as a formula, using powers, roots, exponential, log, sin, cos, etc.)

Does every rational function (a rational function is the quotient of two polynomials) have a simple-form anti-derivative? (Discuss reasons for your opinion with your group.)

What functions would you need to write the anti-derivative of

\[ \int \frac{2x - 1}{x^3 - 3x + 1} \, dx \]

Could you do it without using \( \tan^{-1}(x) \)? Could you do it without using \( \ln(x) \)? Would you need \( e^x \)?