Goals:
(1) Learn to visualize graphs from polar equations
(2) How do parameters affect polar graphs?

Preface  Try first to sketch the graphs by hand, then check them on a calculator or on xFunctions or with each other.

Problem 1.
What shapes occur for each of the following?
For $0 \leq \theta \leq 2\pi$
(1) $r = 1$
(2) $r = 2$
(3) $r = 1/3$
(4) $r = c$ for some constant $c > 0$
(5) $r = 0$
(6) Suppose that $\theta = \pi/4$ and let $r$ vary.

Problem 2. For $0 \leq \theta$
(1) $r = \theta$
(2) $r = 2\theta$
(3) $r = -\theta$
(4) $r = a\theta$ for some constant $a$.
(5) What is true about successive intersections of this curve with the $x$ axis?

Problem 3. For $0 \leq \theta \leq 2\pi$
(1) $r = \theta + 1$
(2) $r = \theta + \pi$
(3) $r = \theta + b$ for some constant $b$
(4) There are two different interpretations of each of these modifications – what are they?

Problem 4. What kind of figures are represented by $r = ae^{\theta+b}$
What is the interpretation of increasing $a$? what is the interpretation
of changing \( b \)? What can you say about successive crossings of the \( x \) axis?

**Problem 5.** Internet search (for after workshops) – what are the names of the figures in 3 and in 4? Which one is related to the nautilus shell?

**Problem 6.** For \( 0 \leq \theta \leq 2\pi \)

(1) \( r = \sin \theta \)

(2) \( r = 2 \sin \theta \)

(3) \( r = a \sin \theta \) for some constant \( a > 0 \). What if \( a < 0 \)?

(4) \( r = 1 + \sin(\theta) \)

(5) \( r = 2 + \sin(\theta) \)

(6) \( r = 1/2 + \sin(\theta) \)

(7) \( r = a + b \sin(\theta) \)

(8) What happens if you \( \sin \theta \) by \( \cos \theta \)?

(9) What happens if you replace \( \theta \) by \( \theta + c \)? How is this related to the item above?

(10) Can you make any general statements about changing \( r = f(\theta) \) by \( r = f(\theta + c) \). Consider problem 3.

**Problem 7.** Consider the function \( r = \frac{\sin(\theta)}{n} + 1 \) for \( n > 1 \). What is the area of this region? What happens to the area as \( n \) goes to infinity?. Does this make sense visually?

**Problem 8.** For \( 0 \leq \theta \leq 2\pi \)

(1) \( r = \sin(2\theta) \)

(2) \( r = \sin(4\theta) \)

(3) \( r = \sin(3\theta) \)

(4) \( r = \sin(5\theta) \)

(5) Describe \( r = \sin(n\theta) \) for integers \( n > 0 \).

(6) What new function would you use to obtain this last graph rotated by by an angle of \( \pi/6 \) radians? Is that clockwise or counter-clockwise?

Web reference http://mathworld.wolfram.com/topics/PolarCurves.html