Goals:

(1) To understand how to manipulate infinite series.

(2) To get a feeling for the convergence and divergence of infinite series.

(3) To see how infinite series are used in fractal geometry.

Problem 1. What is wrong with the following calculation?

\[
0 = 0 + 0 + 0 + \ldots \\
= (1 - 1) + (1 - 1) + (1 - 1) + \ldots \\
= 1 - 1 + 1 - 1 + 1 - 1 + \ldots \\
= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \ldots \\
= 1 + 0 + 0 + 0 + \ldots \\
= 1
\]

Guido Ubaldus thought that this proved the existence of God because “something was created out of nothing”.

Problem 2. If \(\sum a_n\) is divergent and \(\sum b_n\) is divergent, is \(\sum(a_n + b_n)\) necessarily divergent? give examples.

Problem 3. According to Mandelbrodt, natural processes should be modeled with fractals. To construct the snowflake curve, start with an equilateral triangle with sides of length 1. Step 1 in the construction is to divide each side into three equal parts, construct an equilateral triangle on the middle part, and then delete the middle part (see the figure). Step 2 is to repeat Step 1 for each side of the resulting polygon. This process is repeated at each succeeding step. The snowflake curve is the curve that results from repeating this process indefinitely.

from: http://mathworld.wolfram.com/KochSnowflake.html
(a) Let $s_n$, $\ell_n$, and $t_n$ represent the number of sides, the length of each side, and the total length of the $n$th approximating curve (the curve obtained after Step $n$ of the construction), respectively. Find the formulas for $s_n$, $\ell_n$, and $t_n$.

(b) Show that $t_n \to \infty$ as $n \to \infty$.

(c) Sum an infinite series to find the area enclosed by the snowflake curve. To do this, you can try to find the extra area added at each stage of the construction.

Parts (b) and (c) show that the snowflake curve is infinitely long but encloses only a finite area.

**Problem 4.**

A sequence $\{a_n\}$ is defined recursively by the equation

$$a_n = \frac{a_{n-1} + a_{n-2}}{2}$$

for $n \geq 3$, where $a_1$ and $a_2$ can be any real numbers.

(a) Draw the number line with $a_0$ and $a_1$ somewhere on the line. Then draw $a_2$, $a_3$, and so on.

(b) Let $b_n = a_{n+1} - a_n$, so that $b_0 = a_1 - a_0$. You can represent $b_0$ in your diagram by drawing an arrow from $a_0$ to $a_1$. Do the same for $b_1$, $b_2$, and so on.

(c) Find a rule for computing $b_{n+1}$ in terms of $b_n$.

(d) How can you find $\lim_{n \to \infty} a_n$ in terms of the $b_n$?

(e) Use your answer from part (d) to find $\lim_{n \to \infty} a_n$ in terms of $a_0$ and $a_1$. 