Define a isosceles right tetrahedron with side $s$ to be the solid region bounded by the four planes defined by the equations
\[ x = 0, \quad y = 0, \quad z = 0, \quad \text{and} \quad x + y + z = s. \]
It is the 3-dimensional analog of an isosceles right triangle. Three of its edges have length $s$ and are perpendicular to each other. The others have length $s\sqrt{2}$. We know that its volume is $\frac{s^3}{6}$.

Find the volumes of a regular tetrahedron, a regular octahedron and a regular cuboctahedron (look it up), each having edges of length $s$. Here are some hints that may help you.

- A regular tetrahedron can be obtained from a regular cube by removing four isosceles right tetrahedra.
- A regular octahedron can be obtained from a regular tetrahedron by removing four smaller regular tetrahedra.
- A regular cuboctahedron can be obtained from a regular cube by removing eight isosceles right tetrahedra.

Show that a regular cuboctahedron is the disjoint (or nonoverlapping) union of eight regular tetrahedra and a number (to be determined by you) of isosceles right tetrahedra.