Parametric equations

\[ x = x(t) \quad y = y(t) \]

This describes the motion of a particle in the \( xy \)-plane.

\[ \frac{dx}{dt} = \text{speed in } x \text{-direction} \]

\[ \frac{dy}{dt} = \quad \text{y - direction} \]
The total speed is
\[
\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}
\]

WW 6.3

\[x = 6t - 2t^3, \quad y = 6t^2, \quad 0 \leq t \leq 1\]

Rotate this curve about the x-axis and find the surface area.

\[A = \int_{0}^{1} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt\]
\[
\frac{dx}{dt} = 6 - 6t^2 \\
\frac{dy}{dt} = 12t
\]

\[
\left(\frac{dx}{dt}\right)^2 = 36 - 72t^2 + 36t^4
\]

\[
\left(\frac{dy}{dt}\right)^2 = 144t^2
\]

\[
\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 36 + 72t^2 + 36t^4 = \left(6 + 6t^2\right)^2
\]

\[
A = 2\pi \int_0^1 6t^2 \left(6 + 6t^2\right) \, dt
\]

\[
= 2\pi \left[ \int_0^1 \left(36t^2 + 36t^4\right) \, dt - \int_0^1 \left(12t + \frac{36}{5}t^3\right) \, dt \right]
\]

\[
= 2\pi \left[ \left(12 + \frac{36}{5} - 0\right) = 2\pi \frac{96}{5} + \frac{192\pi}{5} \right]
\]
Polar co-ordinates

\( x \) and \( y \) are rectangular coordinates.

\( r \) and \( \theta \) are polar co-ordinates.

\( r = \text{distance from origin} \)

\[
X = r \cos \theta \\
Y = r \sin \theta \\
x/y = \tan \theta \quad \text{slope of red line}
\]

\[
y = \sqrt{x^2 + y^2} \\
\theta = \arctan \left( \frac{y}{x} \right)
\]
Correct about $\theta$.

$\arctan \left( \frac{y}{x} \right)$ is by definition an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

$\frac{\pi}{2} \cdot \arctan \left( \frac{y}{x} \right) < 0$

Correct formula for $\theta$:

$\theta = \begin{cases} 
\arctan \left( \frac{y}{x} \right) & \text{if } x > 0 \\
\pi + \arctan \left( \frac{y}{x} \right) & \text{if } x < 0 
\end{cases}$
Examples of $n = f(\theta)$

$n = \mathbb{R}$
Example \[ M = \cos \theta \]

<table>
<thead>
<tr>
<th>\theta</th>
<th>0</th>
<th>\pi/4</th>
<th>\pi/2</th>
<th>3\pi/4</th>
<th>\pi</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>1</td>
<td>\sqrt{2}</td>
<td>0</td>
<td>-\sqrt{2}</td>
<td>-1</td>
</tr>
</tbody>
</table>

This is a circle of radius \( \frac{1}{2} \) centered at \( \left( \frac{1}{2}, 0 \right) \) with points \( \theta = \pi/4 \), \( \pi/2 \), \( 3\pi/4 \), \( \pi \).
Area in polar coordinates

\[ m = f(\theta) \]

\[ \text{area of slice} = \frac{m^2}{2} \Delta \theta \]

This leads to:

\[ \text{area of region} = \int_{\alpha}^{\beta} \frac{m^2}{2} \, d\theta \]
Pumpkin pie elaboration

area of pie = \( \pi r^2 \)
= \( 36 \pi \)

area of each piece
= \( \frac{36 \pi}{8} = \frac{9 \pi}{2} \)

area of slice
= \( \frac{36 \pi}{2 \cdot 4} = \frac{36 \pi}{8} \)
Graph the corresponding function in rectangular $x$-$y$-plane.

\[ y = 5 \sin 4x \]
This is an 8-leaved rosette of radius 5

m = 5 \times \text{min} 40