Fundamental Theorem of Calculus

If \( f(x) \) is a continuous function for \( a \leq x \leq b \), then the function defined by

\[
g(x) = \int_a^x f(t) \, dt
\]

is continuous and differentiable with

\[
g'(x) = f(x).
\]

Recall \( \int_a^b f(t) \, dt \) is defined in
terms of Riemann sums and is the area under the curve defined by \( f(x) \).

Antiderivatives are harder to find than derivatives!!
Examples
1. \( f(x) = \sqrt{1 + x^3} \)
   Its antiderivative exists but cannot be described in terms of familiar functions.
2. \( f(x) = e^{-x^2} \)