Math 162, Calculus IIA  Extra Credit 5  Name:

Please submit a solution to your instructor by Friday, October 15 before 5 pm.

1. ((1/N+1) points) Figuring out how many spheres can be packed into a container is an important problem with applications to such things as cell biology and the airline industry (not to mention the bowling industry!). A central problem is to find the density of the sphere packing, i.e., the ratio of the volume of the tightest sphere packing to the volume of the container. We propose the following simplified version of the sphere packing problem. Suppose that circles of equal diameter are packed tightly in \( n \) rows inside an equilateral triangle. (See, for example, page 762 [791 in the 5th edition] in your textbook.) If \( A \) is the area of the equilateral triangle and \( A_n \) is the total area occupied by the \( n \) rows of circles, show that

\[
\lim_{n \to \infty} \frac{A_n}{A} = \frac{\pi}{2 \sqrt{3}}.
\]