Hence, trick for summing integers

\[ 1 + 2 + 3 + \cdots + 99 + 100 \]

\[ 100 + 99 + 98 + \cdots + 2 + 1 \]

\[ 100 + 101 + 101 + \cdots + 101 + 101 = 10100 \]

Correct answer = \[\frac{10100}{2} = 5050\]

The sum of integers \( 1 \) through \( n \) is \( \frac{n(n+1)}{2} \).
Arc Length

Mauritius

Sum of blue lengths = first approx. to
lengths of coast line = 210 km

Sum of green lengths = second approx
= 285 km

Sum of red lines = third approx
= 325 km
Question: What is the limit as the number of points goes to \( \infty \)?

**Answer:** This limit is infinite.

\[
y = f(x), \quad a \leq x \leq b
\]

Find length of curve

Divide \([a, b]\) into \( n \) equal subintervals

Sum of red lengths \( \approx \) approximate length of curve
Take the limit as $n \to \infty$.

$$\Delta x = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Take the limit as $\Delta x \to 0$.

Recall $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$. We get

$$\lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$$

$$\frac{dx}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$
Here \( s(x) \) is the arc length function.

\[
s = \int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

Example where the integral is difficult. Find the arc length.
If an ellipse:

\[ y = \pm b \sqrt{1 - \frac{x^2}{a^2}} \]

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

The resulting integral cannot be expressed in terms of familiar functions.

Find the arc length for \( y = \sqrt{x^2 - x} \)

with \( 0 \leq x \leq M \)

circle of radius \( \frac{M}{2} \)

length = \( \frac{\pi M}{2} \)
\[ A = \int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

\[ y' = \frac{-x}{\sqrt{m^2 - x^2}} \]

\[ \sqrt{m^2 - x^2} = \frac{m}{\sqrt{m^2 - x^2}} \]

\[ x = \frac{m \cos \theta}{\sqrt{m^2 - x^2}} \]

\[ \sin \theta = \frac{x}{\sqrt{m^2 - x^2}} \]

\[ \cos \theta = \frac{m}{\sqrt{m^2 - x^2}} \]

\[ \int_0^{\pi/2} d\theta = \frac{\pi}{2} \frac{m}{2} \]

\[ \text{RIGHT ANSWER} \]
\[ Y' = \frac{x}{\sqrt{y^2 - x^2}} \]

\[ 1 + (Y')^2 = 1 + \frac{x^2}{y^2 - x^2} = \frac{x^2 + y^2 - x^2}{y^2 - x^2} = \frac{y^2}{y^2 - x^2} \]

\[ \sqrt{1 + (Y')^2} = \frac{y}{\sqrt{y^2 - x^2}} \]