NAME (please print legibly): __________________________________________
Your University ID Number: ________________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Juan Ortiz-Navarro</td>
<td>MWF 10:00 - 10:50 AM</td>
</tr>
<tr>
<td>Scott Bailey</td>
<td>MWF 9:00 - 9:50 AM</td>
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- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- You are responsible for checking that this exam has all 8 pages.

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Part A

1. (15 points) Consider the polar function

\[ r(\theta) = 1 + \sin(2\theta) \]

(a) Sketch the graph of the “two-paddled fan” on the provided axes.
(b) Set up (but do not evaluate) the integral representing the area of one paddle of the fan.

(c) Set up (but do not evaluate) the integral for the perimeter of one paddle of the fan.
2. (25 points) Consider the graph of the cycloid given parametrically by

\[
\begin{align*}
  x(t) &= 2(t - \sin t) \\
  y(t) &= 2(1 - \cos t)
\end{align*}
\]

(a) What is the area under one arch of the cycloid?

(b) What is the length of one arch of the cycloid?

\textbf{Hint:} \quad 1 - \cos t = 2 \sin^2 \left( \frac{t}{2} \right).

(c) Find the equation of the line tangent to the cycloid at \( t = \frac{\pi}{3} \).
3. (15 points) Consider the sequence whose $n$-th term is $a_n = ne^{-n}$.

(a) Determine if the sequence is increasing, decreasing, or not monotonic.

(b) Is the sequence bounded?

(c) Is the sequence convergent? If it is, what is the limit?
4. (15 points) Determine if the following series are convergent, and if they are, find their sum.

(a) \[ \sum_{n=1}^{\infty} \frac{e^n}{3^n - 1} \]

(b) \[ \sum_{n=1}^{\infty} \ln \left( \frac{n}{n+1} \right) \]
5. (15 points)

(a) Does the following series converge? Why or why not?
\[ \sum_{n=1}^{\infty} n^2 e^{-n^3} \]

(b) If it does, how big is the error when using \( s_5 \), the 5-th partial sum of the series, as an approximation to the sum.
6. (15 points) Do the following series converge? Why or why not?

(a) \[ \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 2} \]

(b) \[ \sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1} \]

(c) \[ \sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k} \]