Work problems

Fabio's swimming pool

Density of Koolaid \( \approx 65.2 \text{ lbs/ft}^3 \)
Basic facts

Work (energy) = force \times distance

English units:

1 pound of force is required to lift a mass of 1 pound.

1 ft-lb = the work (energy) needed
to lift 1 pound a distance of 1 foot.

**Newton units**

\[ \text{Force} = \text{mass} \times \text{acceleration} \]

acceleration due to gravity on earth

\[ = 9.8 \text{ m/s}^2 \]
Force exerted by gravity on a mass of 1 kg is $9.8 \text{ kg m/s}^2$.
$1 \text{ kg m/s}^2 = \text{ unit of force} = 1 \text{ Newton}$.

Work = force $\times$ distance

unit of work = $1 \text{ kg m}^2/\text{sec}^2 = 1 \text{ joule}$. 
Work problems

Fabio's swimming pool

Density of Kool-aid = 65.2 lbs/ft$^3$

Green layer of Koolaid has to lifted $x$ ft.
Volume = area \times thickness = 225 \pi dx \text{ ft}^3

Weight = density \times volume = 65.2 \times 225 \pi dx \text{ lbs}
= 46,000 dx \text{ lbs}

Work needed to lift this layer
= force \times distance = weight \times distance
= 46,000 \times dx \text{ ft-lbs}
Total work needed

\[= \int_{2}^{8.5} 46000 x \, dx = 46000 \int_{2}^{8.5} x \, dx\]

\[= 46000 \left[ \frac{x^2}{2} \right]_2^{8.5} = \text{blah ft lbs.}\]
WW 3.5 (from 2009)

density of hot chocolate

\[ S = 1540 \text{ kg/m}^3 \]

\[ x \text{ = distance from bottom of tank} \]

\[ \text{Radius of layer} = \frac{30}{6} = \frac{12}{3} = m \quad m \quad 0 \leq x \leq 5 \]
area of layer = \( \pi M^2 = \frac{100 \pi}{2} \) m\(^2\)

volume = area \times thickness = 100 \( \pi x^2 \text{d}x \) m\(^3\)

mass of layer = volume \times density = \frac{1540 \times 100 \pi x^2 \text{d}x}{9}

= 17,111 \( \pi x^2 \text{d}x \) kg

force needed to lift this layer

= mass \times acceleration
\[
\text{Work needed to hike other loop}: \int_{674}^{746} (6 - x) \, dx = 678.868 \text{ ft-lb}
\]

\[
\text{Distance to be hiked:} \quad \frac{17,111.7 \text{ ft}}{9.8 \text{ m/s}^2} = 167.88 \text{ ft}
\]

\[
N = \frac{167.88 \text{ ft}}{2.42 \text{ ft}} = 69.61 \text{ N}
\]