\[ x_0 = \text{distance between moving space ships} \]

\[ x_0 = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{v^2}{c^2}} \]

\[ v = \text{speed of space ship} \]

\[ x = \text{distance travelled light-second} = \frac{v}{c} \text{ light} \]
Time dilation factor: \( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \beta \)

What is \( \beta \) on a jet plane

\( v = 540 \text{ mph} = \frac{15}{3600} \text{ miles/second} \)

\( \approx 10^{-6} \text{ C} \)

\( \frac{v}{c} \approx 1/10^6 = 10^{-6} \)

\( \text{let } x = \frac{v^2}{c^2} \) so \( \beta = \frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \frac{3}{8} x^2 \)

In our case \( x = 10^{-12} \approx 1/\text{million} \)

\( \beta = 1 + \frac{1}{3} \times 10^{-12} + \ldots \)
Suppose \( \frac{\sqrt{3}}{2} = 0.866 \)

\[
\frac{v^2}{c^2} = \frac{3}{4} \quad \Rightarrow \quad \frac{1}{c^2} = \frac{1}{4}
\]

\( \beta = \sqrt{1 - \frac{v^2}{c^2}} = 2 \)

Mass is affected the same way that time is.

Let \( m_0 = \) mass of space ship

\( M = \beta m_0 = \) relativistic mass
\[ m = M - m_0 = \text{gain mass} \]
\[ = (\beta - 1) m_0 \]

Recall \( \beta = 1 + \frac{1}{2} \frac{v^2}{c^2} + \ldots \)
\[ \beta - 1 = \frac{1}{2} \frac{v^2}{c^2} + \ldots \]

Kinetic energy of spacecraft
\[ E = \frac{1}{2} m_0 v^2 \]

\[ m = m_0 \left( \frac{1}{2} \frac{v^2}{c^2} \right) = \frac{m_0 v^2}{2} \cdot \frac{1}{c^2} = \frac{E}{c^2} \]
So $E = mc^2$ **EINSTEIN'S EQUATION**

Binomial theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

The coefficient of $x^k y^{n-k}$ in $(x + y)^n$ is denoted $\binom{n}{k}$ **“n choose k”**
\[
\binom{20}{2} = \# \text{ of handshakes made by 20 people}
\]

\[
= 19 + 18 + 17 + 16 \ldots + 1 = \frac{20 \cdot 19}{2} = 190
\]

\[
\binom{n}{k} = \frac{n!}{k! (n-k)!} = \frac{n(n-1)(n-2) \ldots (n-k+1)}{k!}
\]

**k factors**
\[ \binom{n}{k} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k!} \]

This makes sense for any \( n \).

The MacLaurin for \((1+x)^n\) is

\[ \sum_{k=0}^{\infty} \binom{n}{k} x^k = 1 + nx + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \ldots \]

What if \( n = -1 \)

\[ \frac{1}{1+x} = \sum (-1)^k x^k \quad \text{geometric series} \]
What if \( n = -2 \) \( \binom{-2}{k} = (-1)^k (k+1) \)

\[
\begin{align*}
\binom{-1/2}{1/2} &= 1/2 \\
\binom{-1/2}{2} &= -5/16 \\
\binom{-1/2}{3} &= -5/16
\end{align*}
\]

In relativity, we need a series for

\[
\frac{1}{\sqrt{1-x}} = (1-x)^{-1/2} = \sum (-1)^k \binom{-1/2}{k} x^k
\]

\[
= 1 + \frac{1}{2} x + \frac{3}{16} x^2 + \frac{5}{16} x^3 + \ldots
\]