Fundamental Theorem of Calculus

If \( f(x) \) is a continuous function for \( a \leq x \leq b \), then the function
\[
g(x) = \int_a^x f(t) \, dt
\]
is continuous and differentiable with \( g'(x) = f(x) \).

\( g(x) \) is the anti-derivative of \( f(x) \).

Recall \( \int_a^b f(t) \, dt \) is the area under
The curve

Finding the anti-derivative can be tricky.
Example 1: $f(x) = \sqrt{1 + x^3}$
The only derivative cannot be expressed in terms of familiar functions.

\[ f(x) = e^{-x^2} \]
$g(x) = x^2 + 5x^3 + 2x$

$g(x) = \frac{x^8}{8} + \frac{5}{4}x^4 + x^2 + C$

For any value of $C$, $g'(x) = f'(x)$.

Some rules for anti-derivatives:

1. Sum rule: $\int (f_1(x) + f_2(x)) \, dx = \int f_1(x) \, dx + \int f_2(x) \, dx$
2. Constant rule: \[ \int c \cdot f(x) \, dx = c \int f(x) \, dx \]

3. Power rule: \[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + \text{const} \]

for \( n \neq -1 \).

These enable to handle any polynomial.
Bad example

\[ f(x) = \frac{1}{x^2} \quad -1 \leq x \leq 1 \]

Find the area under the curve.

\( f(x) \) is not defined for \( x = 0 \), so

FTC does not apply.
\[ \int_{1}^{5} \frac{dx}{x^2} = -\frac{1}{x} \bigg|_{1}^{5} = -1 - 1 = -2 \text{ Nonsense} \]

5. \[ \int_{5}^{\infty} e^x \, dx = e^x \bigg|_{5}^{\infty} = e^5 - e^1 = e^5 - e \]

6. \[ \int_{0}^{1} x^3 \, dx = \frac{x^4}{4} \bigg|_{0}^{1} = \frac{1}{4} - 0 = \frac{1}{4} \]

area under curve

= \frac{1}{4} \text{ area of square}