EXTRA CREDIT PROBLEM 9

We know that the harmonic series,

$$\sum_{n \geq 1} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

diverges.

(a) Suppose that a determined (and immortal) mathematician started summing the terms of this series at the beginning of time, which we will assume to have occurred 15 billion years ago. Assume that she has been adding one new term per second ever since then. Estimate the partial sum that she would have reached by today. Prove that the sum lies between two numbers within one of each other that you specify.

(b) How many years would it take for the sum to reach half that value, and how many years would it take for it to reach twice that value?

(c) The Tower of Hanoi (look it up) is a game played with 8 concentric disks on three pegs. You can buy it in some toy stores, and you can find interactive versions of it online. There is a Hindu legend about the Tower of Brahma (look it up too), which is the same game played with 64 disks. The legend says that this game has been played by monks who have been making one move per second since the beginning of time, and the world will end when the game is finished. How many years do we have left?

(d) What will be the partial sum (to within 1) of the harmonic series when the Tower of Brahma is complete?