1. (20 points) Describe a nonabelian group of order 21. Hint: Consider the group of $2 \times 2$ matrices (under multiplication) of integers modulo 7 of the form
\[
\begin{bmatrix}
  a & b \\
  0 & 1 \\
\end{bmatrix}
\]
with $a \neq 0$.

2. (20 points) Determine the number of subgroups of order 2 in
   a. $C_8 \times C_8$
   b. $C_4 \times C_4 \times C_4$

3. (10 points) Prove that the intersection of two normal subgroups $H_1$ and $H_2$ of $G$ is a normal subgroup.

4. (10 points) Describe the center of every simple
   a. abelian group
   b. nonabelian group
   and prove your answer.

5. (20 points) You are painting tetrahedral blocks and you have $n$ colors to choose from for each of the 4 faces. Use Burnside’s formula to determine the number of distinguishable blocks that can be made in this way. How big does $n$ have to be in order to get 100 distinguishable blocks?