1. (20 points) You are painting cubic blocks and you have \( n \) colors to choose from for each of the 6 faces. How many distinguishable blocks can be made in this way? How big does \( n \) have to be in order to get at least 100 distinguishable blocks?

2. (20 points) Determine the number of subgroups of order 3 in
   a. \( C_{27} \times C_{27} \)
   b. \( C_9 \times C_9 \times C_9 \)
   c. \( C_3 \times C_3 \times C_3 \times C_3 \)

3. (20 points) Describe a nonabelian group of order 57. \textit{Hint:} Consider the group of \( 2 \times 2 \) matrices (under multiplication) of integers modulo 19 of the form
   \[
   \begin{bmatrix}
   a & b \\
   0 & 1
   \end{bmatrix}
   \]
   with \( a \neq 0 \).

4. (20 points) Let \( X \) be the set of ordered triples \((i, j, k)\) where \( i \) and \( j \) are integers ranging from 1 to 4. Let the symmetric group \( S_4 \) act on this set by permuting the integers in the usual way. Describe the orbits of this \( S_4 \)-set. You do not need Burnside’s formula for this.

5. (20 points) How many Sylow 3-subgroups can a finite group \( G \) have if its order is
   (a) 21
   (b) 39
   (c) 51
   (d) 90