Theorem 1. Let $E$ be a Galois extension of a field $F$, and let $G = \text{Gal}(E/G)$. For any intermediate field $K$ with $F \subseteq K \subseteq E$, let $\chi(K) = \text{Gal}(E/K)$. Then

1. $\chi$ is a one-to-one map from the set of all intermediate fields $K$ to the set of all subgroups of $G$.

2. $K = E^{\text{Gal}(E/K)}$, so $E$ is a Galois extension of every intermediate field $K$.

3. $\chi(E^H) = H$ for all $H \leq G$, so $\chi$ is onto.

4. $[E : K] = |\text{Gal}(E/K)|$.

5. $[K : F]$ is the index of $\text{Gal}(E/K)$ in $G$.

6. $K$ is a Galois extension of $F$ if and only if $\text{Gal}(E/K)$ is a normal subgroup of $G$, in which case

$$\text{Gal}(K/F) \cong G/\text{Gal}(E/K).$$

7. For any two intermediate fields $K_1$ and $K_2$, we have $K_1 \subseteq K_2$ if and only if $\chi(K_1) \supseteq \chi(K_2)$, so the lattice of subgroups $H \leq G$ is the lattice of intermediate fields $F \subseteq K \subseteq E$. 