According to spectral sequence

Given a filtration $F \rightarrow X \rightarrow B$, we want to compute $H^*X$ in terms of $H^*B$ and $H^*F$.

Input $E_2^{s,t} = H^s(B; H^t(F))$.

$s+t = m$ $E_{2s}^{s,t}$ is a subquotient of $H^{s+t}X$.

To describe it, note that given a fiber bundle $F \rightarrow X \rightarrow B$ and a map $f$. 
We get a space \( W = \{ (a, x) \in A \times X : f(a) = \pi(x) \} \) pull back.

Let \( B \) be a CW-complex with skeleton \( B^n \hookrightarrow B \).

We get a map \( H^* X \rightarrow H^* X^{(n)} \) for each \( n \) called the kernel of this map \( F^{n+1} H^* X \).

Thus we get a decreasing sequence of subspaces \( H^* X = F^0 H^* X \supseteq F^1 \supseteq F^2 \supseteq \cdots \).

\( H^n X = F^0 H^n X \supseteq F^1 H^n X \supseteq F^2 H^n X \supseteq \cdots \) \( F^n H^n X = 0 \) for filtration.
\[ F^m H^m X \cong \text{key} \ (H^m X \to H^m X^{(m)}) \]

\[ = 0 \]

\[ E_{m,n}^2 = F^m H^{n+1} / F^n H^{n+1} \]

From this data, we can attempt to resemble \( H^* X \).

To get from \( E^* \) to \( E_{*,*}^\infty \):

There are maps \( \text{(differentials)} \)

\[ d_n^* : E_{m,n}^2 \to E_{m,n}^\infty \]

First index is increased by \( n \).

Sum of indices is increased by \( 1 \).

Then \( d_n^* d_m = 0 \) so we have a
cochain complex

$$E_{n+1}^{s,t} = \ker d_n^{s,t} / \text{im} d_n^{s,t}$$

Note: incoming map is trivial if $$s-n=0$$
but going
$$t-n+1=0$$

Hence for $$n>>0$$

$$E_{n+1}^{s,t} = E_{n}^{s,t}$$

$$E_{\infty}^{s,t}$$ is well defined.

Example

$$O(n)/O(n+1) \rightarrow O(n+1)/O(n-1) \rightarrow O(n+1)/O(n)$$

$$S^{n-1} \rightarrow V_{n+1,2} \rightarrow S^n$$

out of orthonormal 2-frames in $$\mathbb{R}^{n+1}$$
\[ d_m \text{ is the only differential that can be nontrivial.} \]

\[ E_2^{p,q} = H_0(M^p; H_{0,m}^n) \]

\[ E_2^{p,q} = \begin{cases} 2 & \text{if } 0 = 0 \text{ or } m \\ & \text{and } q = 0 \text{ or } n - 1 \\ 0 & \text{else} \end{cases} \]

Suppose \( d_m \) is null by \( e(n) \).

If \( e(n) > 0 \) then \( E_n = E_{n+1} = E_{\infty} \) and

\[ H^* M_{m+1,2} = H^* (S^n \times S^{n-1}) \]
If \( e(n) \neq 0 \) then \( E_{n+1}^{0, n-1} = 0 \) and \( E_2^{n, 0} = 2/e(n) \)

\[
H_i^i(V_{n+1, 2}) = \begin{cases} 
  \geq e(n) & \text{for } i = 0, \ldots, n - 1 \\
  \geq 2 & \text{for } i = n \text{ and } i = 2n - 1 \\
  0 & \text{else}
\end{cases}
\]

If \( e(n) = 0 \)

\[
H_i^i(V_{n+1, 2}) = \begin{cases} 
  \geq & \text{for } i = 0, n - 1, n, 2n - 1 \\
  0 & \text{else}
\end{cases}
\]

\( S^n \) can have a nonzero vector field only if \( H^n(V_{n+1, 2}) \geq 2 \), i.e., only if \( e(n) = 0 \)

Hence \( e(2n - 1) = 0 \).
Another way to get at $e(n)$. Recall the reflection map $\mathbb{R}P^n \to O(n+1)$ which induces $\mathbb{R}P^n \to \mathbb{R}P^{n-1}$, quotienting out the space of unit vectors.

\[
\begin{align*}
S^{n-1} & \rightarrow V_{n+1,2} = \mathbb{R}P^n \rightarrow S^n \\
O(n) / O(n-1) & \uparrow \downarrow \\
\mathbb{R}P^n / (\mathbb{R}P^{n-2}) & \uparrow \downarrow \\
S^{n-1} \cup e^n & = V_{n+1,2}
\end{align*}
\]

In $V_{n+1,2}$, the $n$-cell is attached to $S^{n-1}$. 
by a map of degree $e(n)$. It is the boundary map $\partial_n$ in the cellular chain complex, which is $1 + (-1)^{n+1} = 2$ for even $n$ and $0$ for odd $n$. 