Recall we are knowing that $\pi, s' = \pi$. It reduces to

$$\pi \downarrow s' \downarrow \Rightarrow \pi \downarrow s' \downarrow \pi, s' = \pi$$

$$w = (y, y_0) \times (x, x_0) \overset{F}{\longrightarrow} (x, x_0)$$

given $i, g, p$ and $F$ with $p g = F i$

$F$ with $p F = F$ and $F i = g$

**Unique path lifting**

We need to extend $g$ over the rest of $W$. Let $w_0 = (y_0, 0)$

Let $\omega$ be an overlap current field
of $x_0$ and let $V_0 = F^{-1}(U_0) \subseteq W$. Let $W_0$ be a nbhd of $x_0$, $F^{-1}(U_0) \subseteq D \times U$ for a discrete space $D$. One copy of $U$ in $\mathbb{R}$ contains $x_0$. Use that copy to define $F$ on $U_0$.

$W_0 = \{y_0 \delta\}$

$W \supseteq [0, \delta]$ for $y_0 \in N \cap Y$

and $0 < \delta, \epsilon$ such $W_1 = \{y_0 + \delta\}$
We can make a similar argument on \( X_1 = F(w_k) = F(y_0, x_1) \). It has an evenly covered covering over \( D \) with \( N_1 = F^{-1}(U_k) \). This leads to \( N_2 \times [t_1, t_2] \subset W \) on which we can define \( F \). Continuing in this way, we get \( 0 = t_0 < t_1 < t_2 < t_3 \ldots < t_n = 1 \) and make \( N_1 \) of \( y_0 \) so \( F(N_1 \times [t_1, t_2]) \) is evenly covered so we can define \( F \) on \( N_1 \times [t_1, t_2] \). Finally,
ON : [t_{i-1}, t_i] and hence on $N_0 \times I$ where $N_0 = \bigcap_{i=1}^n N_i$.

In a similar way we can extend $F$ uniquely to all of $W$.

**QED**

We have shown that $\Pi_1(S^1) \cong \mathbb{Z}$.

**Abstract nonsense**.

Pointed spaces $\mathcal{K} \\ \Pi_1$ (homotopy)
Given a map \((X, x_0) \to (Y, y_0)\) we get a homomorphism
\[
\pi_1(X, x_0) \xrightarrow{f_*} \pi_1(Y, y_0)
\]
We also have \(\pi_n\) for \(n > 1\) with \(\pi_n(X, x_0)\) in always abelian.

Def: A category \(\mathcal{C}\) *consists of*
1) a collection of objects
2) for each pair of objects
$X, Y$, a set of morphisms

$X \to Y$

These satisfy certain axioms

**Examples**

1. Sets and maps between them
2. $\text{Top}$ (topological spaces) and continuous maps
3. $\text{Ab}$ (Abelian groups) and homomorphisms
Axioms:
1. For each object $X$ there is an identity morphism $1_X$.
2. Given morphisms $X \to Y$ and $Y \to Z$, we get a morphism $X \to Z$.
3. Composition is associative.
4. Composition with $1_X$ behaves as expected.
Let $C$ and $D$ be categories.

A **functor** $F : C \rightarrow D$ consists of:

1. For each object $X$ in $C$, we get an object $F(X)$ in $D$.
2. For each morphism $f : X \rightarrow Y$ in $C$, we get a morphism $F(f) : F(X) \rightarrow F(Y)$ in $D$.

with $F(fg) = F(f)F(g)$. 


A contravariant functor $G$ from $C$ to $D$ assigns to each object $X$ in $C$ an object $G(X)$ in $D$ and for each morphism $f: X \to Y$ in $C$ a morphism $G(f): G(X) \to G(Y)$ in $D$ with $G(f_1 \circ f_2) = G(f_2) \circ G(f_1)$.

Example: Let $k$ be a field.
We have a category $\text{Vect}_k$ whose objects are vector spaces $V$ over $k$ and morphisms are linear maps. Let $V^* = \text{Hom}(V, k)$. This defines a contravariant functor $\text{Vect}_k \xrightarrow{\mathcal{D}} \text{Vect}_k$.

\[ V \xrightarrow{f} W \quad \Rightarrow \quad V^* = \mathcal{D}(V), \quad W^* = \mathcal{D}(W). \]
Example

Let $G$ be a group and $T$ a set. $G$ is a group if and only if there is a bijection between the free abelian group on $T$ and $G$. Let $A$ be an abelian group. Let $X$ be a set. A homomorphism $F:X \to A$ is equivalent to a set morphism.
Given objects $X, Y$ in $C$, let $C(X, Y)$ denote the set of morphisms from $X$ to $Y$.

In our example:

\[ 	ext{Ab} \xrightarrow{F} \text{free abr.}\]
\[ \text{Sets} \]
\[ \text{Ab} \xrightarrow{G} \text{forgetful function} \]
\[ \text{Ab} \left( F(X), A \right) = \text{Set} \left( X, G(A) \right) \]
$F$ is the left adjoint of $G$.

$G$ is the right adjoint of $F$.

Recall $\mathcal{C}(X,Y)$ is the set of morphisms in $\mathcal{C}$ from $X$ to $Y$. It may have additional structure. Examples:

1. In $\mathcal{Ab}$, $\mathcal{Ab}(A,B)$ is itself an abelian group.
2. In $\mathbf{Top}$, $\mathbf{Top}(X,Y)$ has
the compact open topology, so it is an object in $\textbf{Top}$.

Def. A category $\mathcal{C}$ is enriched over $\mathcal{D}$ if $\mathcal{C}(X,Y)$ is also an object in $\mathcal{D}$ s.t.

A morphism $Y \to Z$ in $\mathcal{C}$ induces $\mathcal{C}(X,Y) \to \mathcal{C}(X,Z)$ in a morphism in $\mathcal{D}$. 

is a morphism in $\mathcal{D}$. 
