Recall in the slice $SS$ for $MU^{(4)}$

\[ \Delta_{\{1+8(2k+1)\}} (\mathbb{Z})^k = 0_{2^k} \oplus 0_{2^k} \quad \text{for } k \geq 1 \]

where $\sigma = \text{sign rep}$ for $G = C_8$

\[ \Delta^{(8)} \rightarrow S^{(k+1)k} \rightarrow MU^{(4)} \]

\[ \Delta^{(8)} = M_{2^k1} \times (M_{2^k1}) = X^2 (M_{2^k1}) \times 3 (M_{2^k1}) \]

\[ \prod_{n} MU^{(4)} = 2 \left\{ \mathbb{Z}^i(m_n) : i = 0 \text{ or } 1 \right\} \]

\[ M_n \in \prod_{2i} \mathbb{Z}^i \]
Inverting \( \Delta_k^{(n)} \) makes \( \mu_{2k+6} = \mu_{2k} \)
a permanent cycle

Goal: Invert something to some power of \( M_{256} \) a permanent cycle.

We will define a \( D \in \Pi^G_{1928} \)

\[ S^2 = D^{-1} \cdot MU^{(4)} \]

\[ S^2 \sim 256 - \text{periodic} \]

Since we are inverting an elt in \( \Pi^G_{mp_8} \)
the resulting telescope has the gap property, i.e. \( \text{Tr} \, \mathcal{L}^g = 0 \) for \(-4 < k < 0\).

Lemma. Let \( H \subseteq G = G_0 \) and \( V \) an \( \text{irrep} \) of \( H \). Then \( U_v \in \prod_{V' \in H} H \).

\[
\begin{align*}
0 & \quad \Rightarrow \quad S_{IV} \quad \Rightarrow \quad S_{IV^*} \\
\text{Then} \quad & \quad M_{V''} = N^G_H(U_v) = M_{V'} \\
& \quad \text{where} \quad V' = \text{ind}_H^G V \quad \text{and} \quad V'' = \text{ind}_H^G IV
\end{align*}
\]

Proof: Applying norm functor to \( D \) gives
Consider $V = \mathbb{R}P^4 = \text{reg rep of } C_4$

\[
\begin{align*}
M_{2s^6} & = N_{4}^{8 \left( M_{2s^4} \right) M_{6s}} \quad \xi = \text{sign rep of } C_4 \\
M_{2s^4} & = N_{2}^{4 \left( M_{2s} \right) M_{4s^4}} = N_{2}^{4 \left( M_{2s^2} \right) M_{4s^4}} \\
N_{4}^{s \left( M_{2s^2} \right) M_{4s^4}} & = N_{2}^{8 \left( M_{2s^2} \right) M_{4s^4}} \quad N_{4}^{8 \left( M_{4s^4} \right)}
\end{align*}
\]
\[ M_{2p_6} = N^8 \langle u_{2p_6} \rangle N^8 \langle u_{48} \rangle M_{2p_6} \]

We make a power of \( N^8 \langle u_{2p_6} \rangle \) a perm cycle by inverting \( \Delta_{k}^{(4)} \) for some \( k \).

We can a power of \( M_{2p_6} \) a perm cycle by inverting \( N^8 \langle \Delta_{k_1}^{(3)} \rangle \) and \( N^8 \langle \Delta_{k_2}^{(4)} \rangle \) \( \Delta_{k_3}^{(8)} \).

We need to choose \( k_m \) so that the resulting \( S_2 \), detects the \( \text{O}_j \) s.

It turns out that we need \( 8 \mid 2^m k_m \).
for \( m = 1, 2, 3 \). (To be explained later)

Let \( k_1 = 4 \), \( k_2 = 2 \), \( k_3 = 1 \)

Inverting \( \Delta_4^{(2)} \) makes \( u_{3298} \) a perm cycle

\[
\begin{array}{cccc}
\Delta_4^{(2)} & \Delta_2^{(4)} & \Delta_2 & \Delta_1^{(8)} & \Delta_1 & \Delta_1^{(4)} & \Delta_1\end{array}
\]

Let \( D = \Delta_1^{(8)} N_4 (\Delta_2^{(4)} N_2 \Delta_4^{(2)} \cdot C_8 \cdot MV^{(4)}_{1998}) \)

Inverting \( D \) makes \( u_{3298} = u_{298}^{16} \)

a permanent cycle.
Let \( \tilde{S}_2 = D^{-1} \text{MU}(4) \)

\[
\begin{align*}
S^0 & \xrightarrow{D} \Sigma^{-19p_8} \text{MU}(4) \\
\text{MU}(4) & \rightarrow \Sigma^{-19p_8} \text{MU}(4) \\
\text{MU}(4) & \rightarrow \Sigma^{-38p_8} \text{MU}(4) \\
\end{align*}
\]

\( D \) can be iterated

\[
\begin{align*}
\text{MU}(4) & \xrightarrow{D} \Sigma^{-19p_8} \text{MU}(4) \\
\Sigma^{-19p_8} \text{MU}(4) & \xrightarrow{D} \Sigma^{-38p_8} \text{MU}(4) \\
\end{align*}
\]

\( \tilde{S}_2 = \text{colim} \Sigma^{-19p_8} \text{MU}(4) \)

\[
\begin{align*}
\tilde{S}_2 & \xrightarrow{\mathbb{Z}_{256}} \Sigma^{32p_8} \\
\Sigma^{32p_8} & \xrightarrow{1/2} \Sigma \\
\end{align*}
\]

Let \( \Delta_1 = M_{32p_8} (\tilde{S}_2) \in E_2^{0,16} \tilde{S}_2 \)
\((\Delta_1)^{16} = \mu_{32}\Delta_8\) \((\Delta_1)^{32} \in E_2 256 \to \Sigma\)

It is a permutation cycle. We have a map

\[ \sigma: \Sigma_{256} \to \Sigma_2 \]

\(\Delta_1\) is a factor of \(D\).

Restricting to the trivial \(\rho\) makes \(\mu_{32}\Delta_8\) a unit. The map \(\sigma\) induces an equivariant restriction to \(C_1\).
Hence we get an equation

\[ \sum_{256} \Sigma \frac{1}{2} \rightarrow \Sigma \frac{1}{2} \]

This is the periodicity for \( \Sigma \frac{1}{2} \).

Our fixed but thin says:

If \( X \) is a \( G \)-spectrum with \( \text{fr}_G X = \ast \),

then \( X^G = X \).

Then

\[ D = \Delta^{(8)} \ldots \]

\[ \Delta^{(8)} = \gamma_1, \gamma^1(m), \gamma^2(m), \gamma^3(m) \]
\[
\begin{align*}
S^8 & \xrightarrow{\Delta_1^{(8)}} M\nu(c_1) \\
S^4 & \xrightarrow{\Sigma} MO \\
T_1 MO & = 0 \\
\Phi_{c_2}(\Delta_1^{(8)}) & = 0 \\
\Phi_{c_2}(S^2) & = x 
\end{align*}
\]

Hence our fixed point theorem says

\[
S^2 c_6 = S^2 h c_8 
\]

We know
1) \[ 256 \cdot 52^{4c_8} = 2^{4m_c_8} \]

2) The slice SS for \( T \times \{c_8\} \) has the gap property.

3) \( S_2 \) has the detection property.

\[ S_2 = S_2^{4c_8} = S_2^{4c_8} \]
Preview of detection

It is known that the homotopy fixed point is for $\Omega (\mathbb{C} \mathbb{U}(1) \times \mathbb{C})$

\[ E_2 = H^* (C_8 \times S^2) \]

We have $\delta_1 E_2^{2^{n+1}} (S^0)$ in the ANSS for $S^0$.
We will map $H^*(C_8; \mathbb{Z}_2)$ to a simpler gadget related to a certain formal $A$-module

where $A = \mathbb{Z}_2[\sqrt{5}]/(\sqrt{5} + 1)$

In $A$, let $\pi = 5 - 1$

$A/\langle \pi \rangle \cong \mathbb{Z}/2$, i.e. $\langle \pi \rangle$ is the maximal ideal and $\pi^4 = 2$. unit. There is a FGL $A[\pi^\infty]$ with $\log x = \sum_{n=0}^{\infty} \frac{x^{2^n}}{\pi^n} (\pi - 1)$
D will map to an easily \( H^2(?) \) related to this FGL.