Extra meetings
Tomorrow 3:30 here
Friday 4/30 4:00 Nylan 101

Toward the detection theorem
Structure of $BP_*BP = BP_* [x_1, x_2, \ldots : J$

$BP_* \cong \mathbb{Z}(\phi) \oplus [V_1, V_2, \ldots : J$

$|W_n| = |I_n| \geq 2(\beta^n-1)$

3 facts about this structure
1) $\Delta (x_i) = x_i \otimes 1 + 1 \otimes x_i$
2) \[ \eta_\alpha (w_1) = \nu_1 + pt \]

3) \[ \eta_\alpha (w_2) = \nu_2 + \nu_1 \cdot \frac{1}{p} - \nu_1 \cdot pt \mod p \]

How to construct some elements in \( \text{Ext} = \text{Ext}^1_{BP_\times} (BP_\times, BP_\times) \)?

a) Consider the short exact seq

\[ 0 \to BP_\times \to \beta^{-1}BP_\times \to \frac{BP_\times}{p} \to 0 \]

\[ \text{BP}_\times \otimes A \]

\[ 0 \to \mathbb{Z}(p) \to A \to A / \mathbb{Z}(p) \to 0 \]

\[ \lim_\to_{\to} \geq \frac{1}{p} \]
In \( BP_\ast / p^\infty \) we have \( \frac{v_i}{p} \), which is invariant, i.e., \( \eta_L(\frac{v_i}{p}) = \eta_R(\frac{v_i}{p}) \).

\[
\eta_R\left(\frac{v_i}{p}\right) = \frac{(v_1 + \text{ht}_1)^i}{p} = \frac{v_i^i}{p} = \eta_R\left(\frac{v_i^i}{p}\right)
\]

This means it is in \( \text{Ext}_p^0 \left( BP_\ast / p^\infty \right) \).

\[0 \to BP_\ast \to p^{-1}BP_\ast \to BP_\ast / p^\infty \to 0\]

We get a long exact seq of \( \text{Ext}_p^i \).
\[ \begin{align*}
\mathbb{Z}[\tau] & \\
\mathbb{Q} & \\
\mathbb{Q}/\mathbb{Z}[\tau] & \text{other stuff} \\
0 & \to \text{Ext}^0 & \to \text{Ext}^0(\mathbb{A}^{-1}BP_\infty) & \to \text{Ext}^0(BP_\infty/\mathbb{A}^\infty) \\
\text{other stuff} & \\
\text{Ext}^i & \to \text{Ext}^i(\mathbb{A}^{-1}BP_\infty) & \to \cdots \\
\alpha_i & \text{other stuff} & \\
0 & \\
\end{align*} \]

for \( i > 0 \), \( \alpha_i \in \text{Ext}^i \)

There is another SES

\[ \begin{align*}
0 & \to BP_\infty/\mathbb{A}^\infty & \to N_{-1}BP_\infty/\mathbb{A}^\infty & \to BP_\infty/(\mathbb{A}^\infty, \nu^\infty) & \to \nu
\end{align*} \]
and is related to \( \Omega^{n+1} \).

We need to show this element \( \phi \) maps nontrivially to a similar element \( \phi' \) for the spectrum \( S' \).

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Toward the slice theorem

The slice theorem describes the layers of the slice tower for \( MU(\mathbb{Q}/\mathbb{Z}) \):

\[ g = G = C_{2^n} \]
The slice tower is an equivariant analog of the classical Postnikov tower.

\[ \varphi: X \rightarrow \mathbb{P}^n \times X \] is the map obtained by killing all fibers above dimension \( n \).

Its fiber \( \mathbb{P}^n \times X \) is the \( n \)-connected cover of \( X \).

In the equivariant analog, we replace spheres by the
following: \[ H \leq G \quad \rho_n = \text{reg rep of } H \]

\[ \prod_{m \geq 2} \tilde{S}(m\rho_n) = \bigcap_{H}^{\text{H}} S^{m\rho_n} = \text{wedge of } g/\text{H copies of } S^{m\rho_n} \]

and \[ \prod_{m \geq 2} \tilde{S}(m\rho_n) \]. These are called slice cells. We get a tower

\[ \begin{array}{ccc}
\vdots & \to \mathbb{C} \mathbb{P}^n & \to \mathbb{C} \mathbb{P}^{n-1} & \to \vdots \\
& & (m \text{-th slice}) & \\
& & \text{the } n-\text{th layer is the fiber of}
\end{array} \]

the map shown above
\[ \left[ \mathcal{S}(m^p_n), \mathcal{G}P^n \right] = 0 \quad \text{if} \quad mn > n \]

\[ \left[ \mathcal{Z}^{-1}, \mathcal{S}(m^p_n), \mathcal{G}P^n \right] = 0 \quad \text{if} \quad mn - 1 > n \]

The Alice Theorem identifies

\[ \mathcal{G}P^m \bigotimes \mathcal{M}V(g/2) \]

It is contractible for \( m \) odd

wedge of \( \mathcal{S}(m^p_n) \wedge \mathcal{H} \) for \( m \) even. The trivial subgroup of \( \mathcal{G} \)

never occurs.