Steenrod operations can be iterated. For a sequence of nonnegative integers
$I = (i_1, i_2, \dots, i_r)$, let $Sq^I = Sq^{i_1} Sq^{i_2} \dots$.
We will say that I is *admissible* if $i_k \geq 2i_{k+1}$ for each k. Define
$n(I) = i_1 + i_2 + \cdots$ and the excess
$e(I) = 2i_1 - n(I) = (i_1 - 2i_2) + (i_2 - 2i_3) + \cdots + (i_{r-1} - 2i_r) + i_r$.

Suppose that $H^*F = P(z_1, z_2, \dots)$ with $n_i = |z_i|$. Let $L(a,r) = (2^{r-1}a \dots, 2a, a)$.
Note that $n\big(L(a,r)\big) = (2^r - 1)a$ and $e\big(L(a,r)\big) = a$.
Then $Sq^{L(n_i, r)}(z_i) = z_i^{2^r}$.
A simple system of generators for H^*F is
$\{Sq^{L(n_i,r)}(z_i) : i = 1,2, \dots; r \geq 0\}$.

Recall $K_n = K(Z/2, n)$. $H^*(K_1) = P(u_1)$ where $u_1 \in H^1$.
$H^*(K_2) = P(x_2, x_3, x_5 \dots.) = P\big(u_2, Sq^1 u_2, Sq^{(2,1)} u_2, \dots\big)$
$= P(Sq^I(u_2): I \text{ is admissible}, e(I) < 2)$.

Theorem. In general we have $H^*(K_n) = P(Sq^I(u_n): I \text{ is admissible}, e(I) < n)$.

Combinatorial Lemma. Let $J = (j_1, j_2, \dots, j_k)$ be admissible with $e(J) < n - 1$. Let
$s_J = n - 1 + n(J)$. For each $r \geq 0$ let $I = \big(2^{r-1}s_J, 2^{r-2}s_J, \dots, s_J, j_1, j_2, \dots j_k\big)$. Then every admissible sequence with excess <n has this form.

Proof: Note $s_J - 2j_1 = n - 1 + n(J) - 2j_1 = n - 1 - e(J) > 0$, so I is admissible.
If $r = 0$, then $I = J$.
For $r > 0$, $e(I) = \big(s_J - 2j_1\big) + e(J) = n - 1 - e(J) + e(J) = n - 1$.

Conversely let $I = (i_1, i_2, \dots i_\ell)$ have excess $n - 1$. Let r be the largest integer
such that $i_1 = 2i_2 = 4i_3 \dots = 2^{r-1}i_r$ and $J = (i_{r+1}, \dots i_\ell)$.
Then $e(I) = (i_r - 2i_{r+1}) + e(J) = (i_r - 2i_{r+1}) + 2i_{r+1} - n(J) = i\_r - n(J) = n - 1$
So $i_r = n - 1 + n(J) = s_J$.

Then I has the desired form. QED.

Let E denote the spectrum whose nth is K_n. This is the mod 2 Eilenberg-Mac Lane spectrum.
Then $H^*(E) = Z/2\{ Sq^I(u): I \text{ admissible}\}$.

Adem relation: For $a < 2b$, $Sq^a Sq^b$ is a linear combination of admissible products.

Define the mod 2 Steenrod algebra A to be the free associative algebra generated by the Sq^i fgor i>0, smodule the Adem relations. The admissible monomial form a basis of it.

Then H^*X for any space or spectrum X has a natural A-module structure, and $H^*E = A$.

Let X and Y be pointed spaces. Then their smash product $X \wedge Y$ is $(X \times Y) / (X \vee Y)$. $S^m \wedge S^n = S^{m+n}$. What about the smash product of spectra? Suppose X is the suspension spectrum of a space X, and Y is a spectrum. Then we can define the spectrum $W = X \wedge Y$ by $W_n = X \wedge Y_n$.

Basic facts about the mod 2 Eilenberg-Mac Lane spectrum E.

Recall that for a space X, $H^n(X; Z/2) = [X, K_n]$, the set of homotopy classes of maps $X \to K_n$.
For a spectrum X, $H^n(X; Z/2) = [X, \Sigma^n E]$.
$H_n(X; Z/2) = \pi_n(X \wedge E)$. The functor on the right satisfiues the Eilenberg-Steenrod axioms.