Recall we have topological G-categories categories $\mathcal{T}_G$ (formerly $\underline{\mathcal{T}_G}$), the category of pointed G-spaces and $\mathcal{J}_G$, the category of finite dimensional orthogonal representations V of G with $\mathcal{J}_G(V,W)$ being the Thom space of the vector bundle over $O(V,W)$, the sapce of orthogonal embeddings f of V into W, where the fiber over f is a the orthogonal complement of V in W as embedded by f.

If dim V>dim W, $\mathcal{J}_G(V,W) = *$. If dim V= dimW, $\mathcal{J}_G(V,W) = O(V,W)_+$. If dimV<dimW, then $\mathcal{J}_G(V,W)$ is connected and noncontractible. $\mathcal{J}_G(0,W) = S^W$. Composition of morphisms is given by $\mathcal{J}_G(V,W) \wedge \mathcal{J}_G(U,V) \to \mathcal{J}_G(U,W)$. When V=U or W we get get a right action of $O(U)$ and a left action of $O(W)$ on $\mathcal{J}_G(U,W)$.

A G-spectrum X is a functor $\mathcal{J}_G \to \mathcal{T}_G$. We denote its value on V by X_V. We have structure maps $\mathcal{J}_G(V,W) \wedge X_V \to X_W$. In particular X_V has a left action of $O(V)$. The structure map must factor through the orbit space $\mathcal{J}_G(V,W) \wedge_{O(V)} X_V$.

Recall the tautological presentation of X, $\text{colim}_V\, S^{-V} \wedge X_V$, where S^{-V} is the spectrum defined by $(S^{-V})_W = \mathcal{J}_G(V,W)$ and $(S^{-V} \wedge X_V)_W = (S^{-V})_W \wedge X_V$. The colimit is short had for a certain coequalizer. See previous notes. In particular S^{-0} is the sphere spectrum. We reserve ther symbol S^0 for the usal space.

<u>Smash products</u>. We can define the smash product of two spectra by something similar to the tautological presentation, $X \wedge Y = colim_{V,W}\, S^{-V \oplus W} \wedge X_V \wedge Y_W$.

Alternate description: We have a functor $X \wedge' Y\colon \mathcal{J}_G \times \mathcal{J}_G \to \mathcal{T}_G$ by $(V,W) \mapsto X_V \wedge Y_W$. The external smash product. We also have $K\colon \mathcal{J}_G \times \mathcal{J}_G \to \mathcal{J}_G$ defined by $(V,W) \mapsto V \oplus W$. We define the functor $X \wedge Y$ to be the left Kan extension of $X \wedge' Y$ along K.

Facts: $S^{-V} \wedge S^{-W} = S^{-V \oplus W}$. $X \wedge S^{-0} = S^{-0} \wedge X = X$ for any spectrum X.

This makes $\mathcal{S}^G$ into a symmetric monoidal category.