Recall that $S_G$ denotes the topological $G$-category of orthogonal $G$-spectra and nonequivariant maps, that is functors $J_G \to T_G$, while $S^G$ denotes the topological category of orthogonal $G$-spectra and equivariant maps. We use the symbols $\text{Comm}_G$, $\text{Alg}_G$, $\text{Comm}^G$ and $\text{Alg}^G$ to denote the categories of commutative and associative algebras (under smash product) within these two categories. They are bicomplete, and the evident forgetful functors have left adjoints of the form

$$X \mapsto \text{Sym}(X) = \vee_{k \geq 0} \text{Sym}^k(X) \quad \text{and} \quad X \mapsto T(X) = \vee_{k \geq 0} X^{(k)}.$$ 

One can define left and right modules $M$ over such algebras $A$. The left modules form a category $\mathcal{M}_A$. When $A$ is commutative, the category $\mathcal{M}_A$ inherits a symmetric monoidal product $M \wedge_A N$ defined by the reflexive coequalizer diagram

$$M \wedge A \wedge N \rightrightarrows M \wedge N \to M \wedge_A N.$$ 

Let $H \subset G$ be a subgroup of index $m$. We have seen (3/31) that there is a norm functor $N^G_H : S^H \to S^G$ that is right adjoint to the forgetful functor. We also have $N^G_H : \text{Comm}^H \to \text{Comm}^G$ that is left adjoint to the forgetful functor.

We know what a weak equivalence of $G$-spectra is, namely a map that induces an isomorphisms of homotopy groups on all of the fixed point sets. It is less clear what the fibrations and cofibrations should be. There is a theory that helps here, namely that of homotopical categories, not to be confused with homotopy categories. Roughly speaking they are categories with weak equivalences waiting for model structures. They were introduced in 2004 by Kan and 3 of his former students in *Homotopy Limit Functors on Model Categories and Homotopical Categories*, and are treated in Chapter 2 of Riehl's book, *Categorical homotopy theory*.

Def. A homotopical category $\mathcal{M}$ is a category with a wide (includes all object and all isomorphisms) subcategory $\mathcal{W}$ satisfying the 2-of-6 condition: Given a diagram

$$W \to X \to Y \to Z$$

In $\mathcal{M}$, with $gf$ and $hg$ are in $\mathcal{W}$, then so are $f$, $g$, $h$, and $hgf$.

Its homotopy category $Ho(\mathcal{M})$ is obtained by formally inverting all weak equivalences. There is a functor $\gamma : \mathcal{M} \to Ho(\mathcal{M})$. A functor $F : \mathcal{M} \to \mathcal{N}$ between homotopical categories may or may not preserve weak equivalences.

Def. (2.2.1 of Riehl) A left deformation on a homotopical category $\mathcal{M}$ is an endofunctor
$Q : \mathcal{M} \to \mathcal{M}$ with a natural transformation $q : Q \Rightarrow 1$. Let $\mathcal{M}_Q$ be the subcategory of objects defined by $Q$, the "cofibrant" objects.

Example. Let $\mathcal{M}$ be a model category with functorial cofibrant replacement $Q$.

Def. (Riehl 2.2.4) A left deformation for a functor $F : \mathcal{M} \to \mathcal{N}$ between homotopical categories is a left deformation $Q$ such that $F$ is homotopical on the subcategory $\mathcal{M}_Q$. If such exists, we say $F$ is left deformable.

Def (2.2.1 of Hovey) Let $\mathcal{C}$ and $\mathcal{D}$ be model categories. A functor $F : \mathcal{C} \to \mathcal{D}$ is left Quillen if it is a left adjoint and preserves cofibrations and acyclic cofibrations. A right Quillen functor is .... A pair of adjoint functors

\[
\begin{array}{ccc}
F & \& \text{and} \\
\circ & \varepsilon & \circ \\
\mathcal{C} & \xrightarrow{\varepsilon} & \mathcal{D} \\
\end{array}
\]

Is a Quillen adjunction if $F$ is left Quillen. (This makes $G$ a right Quillen functor.)

Example:

\[
\begin{array}{ccc}
\text{Set} & \xleftarrow{\text{Top}} & \text{Cling} \\
\end{array}
\]

Ken Brown's Lemma. A left (right )Quillen functor preserves weak equivalences of cofibrant (fibrant) objects.

When a functor $F$ is left deformable, there is a maximal subcategory on which it is homotopical.