Every model category $\boldsymbol{C}$ has a *homotopy category* $\mathrm{Ho}(\boldsymbol{C})$ obtained from $\boldsymbol{C}$ by formally inverting all weak equivalences. Its objects are those of $\boldsymbol{C}$, and its morphisms are those of along with inverses of weak equivalences. Hence weak equivalences in $\boldsymbol{C}$ become isomorphisms in $\mathrm{Ho}(\boldsymbol{C})$. There is a functor $\gamma: \boldsymbol{C} \to \mathrm{Ho}(\boldsymbol{C})$ that is the identity on objects. More details will be given later.

We can ask to what extent a functor $F: \boldsymbol{C} \to \boldsymbol{D}$ from a model category $\boldsymbol{C}$ can be factored throught the homotopy category $\mathrm{Ho}(\boldsymbol{C})$ obtained from $\boldsymbol{C}$ by formally inverting all weak equivalences. We have a functor $\gamma: \mathbf{C} \to \mathrm{Ho}(\boldsymbol{C})$. Now consider pairs (G, s) where $G: \mathrm{Ho}(\boldsymbol{C}) \to D$ and s is a natural transformation from $G\gamma$ to F. A *left derived functor* (LF, t), if it exists, is such a pair with the universal property that any such pair (G, s) admits a unique natural transformation g to LF such that $s = t\ g$. In other words it is a *right* Kan extension of F along γ.

Similarly a *right derived functor* $RF: \mathrm{Ho}(\boldsymbol{C}) \to D$ is a left Kan extension of F along γ.
In both cases the universal property implies uniqueness. It is customary to omit the natural transformation from the notation.

It can be shown (Prop 9.3 of Dwyer-Spalinski) that a functor F on a model category $\boldsymbol{C}$ which converts weak equivalences between cofibrant objects to weak isomorphisms has a left derived functor LF which agrees with F on cofibrant objects.

Now suppose that $F: \boldsymbol{C} \to \boldsymbol{D}$ is a functor between model categories. A total left (right) derived functor $\mathbf{L}F$ $(\mathbf{R}F)$ is a left (right) derived functor (and hence a right (left) Kan extension) for the composite $\gamma_{\boldsymbol{D}}F$.

This notion of a derived functor is related to the one in homological algebra in the following way. For a ring R let $\mathbf{Ch}_R$ denote the category of nonnegatively graded chain complexes of left R-modules. It has a model structure in which weak equivalences are maps inducing isomorphisms in homology (quasi-isomorphisms), fibrations are surjections, and cofibrations are injections with projective cokernel in each degree. This means that the cofibrant objects are chain complexes of projective R-modules. For an R-module N, let $K(N, 0)$ denote the chain complex which is N concentrated in degree 0. It has a cofibrant replacement $P \to K(N, 0)$ where P is a projective resolution of N.

For a right R-module M, the functor $M \otimes -$ defines a functor $F: \mathbf{Ch}_R \to \mathbf{Ch}_{\boldsymbol{Z}}$. It has a total left derived functor $\mathbf{L}F: \mathrm{Ho}(\mathbf{Ch}_R) \to \mathrm{Ho}(\mathbf{Ch}_{\boldsymbol{Z}})$**.** Then it follows from the above that there is a natural isomorphism

$$H_i \mathbf{L}F\big(K(N, 0)\big) \cong \mathrm{Tor}_i^R(M, N) \text{ for all } i \geq 0.$$