Recall the mod 2 Steenrod algebra A

It is an associative generated elements Sq^i for $i>0$ ($Sq^0=1$) subject to Adem relations. For $a<2b$

$$Sq^a Sq^b = \sum_{i\geq 0} \binom{b-1-i}{a-2i} Sq^{a+b-i} Sq^i$$

Def A monomial $Sq^{i_1} Sq^{i_2} \cdots Sq^{i_n}$ is ADMISSIBLE if $i_k \geq 2 i_{k+1}$

Sq^I

$I=(i_1, i_2, \ldots, i_n)$

Thm The admissible monomials form a basis for A.

Def For an admissible I as above, its EXCESS $e(I) = (i_1 - 2i_2) + (i_2 - 2i_3) \cdots + (i_n)$

$= i_1 - i_2 - i_3 - i_4 \cdots - i_n$

Recall for $x \in H^n(X)$, then $Sq^i x = \begin{cases} 0 & \text{if } i>n \\ x^2 & \text{if } i=n \\ ? & \text{if } i<n \end{cases}$

$$Sq^{i_1} Sq^{i_2} x = \begin{cases} 0 & \text{if } i_2 > n \\ Sq^{i_1} x^2 & \text{if } i_2 = n \\ Sq^{i_1} Sq^{i_2} x & i_2 < n \end{cases}$$

$= \{ \ 0 \quad$ if $i_2 > n$

$$= \begin{cases} 0 & \text{if } i_2 > n \\ 0 & \text{if } i_2 = n \text{ and } i_1 > 2n \\ x^4 & \text{if } i_2 = n \text{ and } i_1 = 2n \\ 0 & \text{if } i_2 < n \text{ and } i_1 > n + i_2 \end{cases}$$

$e(i_1, i_2) = i_1 - i_2$

Claim that $Sq^{i_1} Sq^{i_2} x = 0$ if $e(i_1, i_2) > n$

If $i_2 > n$, then $e(I) = (i_1 - 2i_2) + i_2 \geq i_2 > n$

If $i_2 = n$ and $i_1 > 2n$, then

$$e(I) = (i_1 - 2i_2) + i_2 > n$$

If $i_1 = 2n$ and $i_2 = n$ then $e = (2n - 2n) + n = n$

If $i_1 > n + i_2$ then $e = i_1 - 2i_2 + i_2 = i_1 - i_2 > n$

If $i_1 = n + i_2$ then $e = n$ and $Sq^{i_1} Sq^{i_2} x = (Sq^{i_2} x)^2$

Prop For $x \in H^n$, $Sq^I x = \begin{cases} 0 & \text{if } e(I) > n \\ (\sim)^2 & \text{if } e(I) = n \\ ? & \text{if } e(I) < n \end{cases}$