Note Title 10/10/2012

# Dillon on 0-slices

<u>Thm</u> The category of 0-slices is that of Mackey functors $\underline{M}$ with injective restrictions.

<u>Pf</u> Since $G_+ \wedge_H S^1 \in \mathcal{T}_{\geq 1}$ and $\mathcal{T}_{\geq 0}$ is the category of $(-1)$-connected spectra, we know all 0-slices have the form $H\underline{M}$ for some $\underline{M}$.

① Will show $[S^{\rho_G - 1}, H\underline{M}] = \{x \in \underline{M}(G/G) : \mathrm{Res}^G_H(x) = 0 \ \forall H < G\}$

This implies $[G_+ \wedge_H S^{\rho_H - 1}, H\underline{M}] = \{x \in \underline{M}(H/H) : \mathrm{Res}^H_K(x) \ \forall K < H\}$

These gps must vanish if $H\underline{M}$ is a 0-slice.

To prove ①

$$[S^{\rho_G - 1}, H\underline{M}] = H^0_G(S^{\rho_G - 1}, \underline{M})$$

We need a cell structure for $S^{\rho_G - 1} = X$

$\partial \Delta^{|G|-1}$ has $g = |G|$ vertices permuted by $G$ in $S^{\rho_G - 1}$. This cell structure is not equiv but its barycentric subdivision is. Here a $(k-1)$-simplex is a chain of $k$ proper inclusions of nonempty subsets of $G$. We get a cx

$$\underline{M}(G/G) \to \underline{M}(S_1) \to \underline{M}(S_2) \to \underline{M}(S_3) \to \underline{M}(S_{g-1})$$

where $S_k$ = set of $(k-1)$-simplices

$\underline{M}$ is contravariant map

coboundary is an alternating sum.

$S_1$ = collection of nonempty proper subsets of $G$. Each is stabilized by a proper subgp, e.g. the collection of left cosets of $H$ is a copy of $G/H$. Hence an elt of $H^0$ is in the kernel each $\mathrm{Res}^G_H$ for proper $H$.

This condition on $\underline{M}$ is necessary for $H\underline{M}$ to be a 0-slice. Is it sufficient???

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Let $G = C_2$. We want $\underline{\pi}_* H\underline{\mathbb{Z}}$

Have found $\underline{H}_* S^{n\sigma}$ for each $n \in \mathbb{Z}$

$$\underline{H}_i S^{n\sigma} = \underline{\pi}_i S^{-n\sigma} \wedge H\underline{\mathbb{Z}} = \underline{\pi}_{i-n\sigma} H\underline{\mathbb{Z}}$$

are known for all $i$ and $n$

$$S^{n\rho_2} = S^{n(1+\sigma)} = S^{n+n\sigma}$$

$S^{n\rho_2}$ is a $(2n)$-slice cell for each $n$

$H\underline{\mathbb{Z}} \wedge S^{n\rho_2}$ is a $(2n)$-slice $\forall n$.

The slice tower for $H\underline{\mathbb{Z}} \wedge S^{n\rho_2}$ has has just one layer, the $(2n)$th

$$P_m^m\left(H\underline{\mathbb{Z}} \wedge S^{n\rho_2}\right) = \begin{cases} H\underline{\mathbb{Z}} \wedge S^{n\rho_2} & \text{for } m = 2n \\ * & \text{for } m \neq 2n \end{cases}$$