

Thm A O -slice is an Eilenberg-Mac Lane spectrum $\underline{H}\underline{M}$ where \underline{M} is a Mackey functor with injective restriction maps.

Proof We know this condition on \underline{M} is necessary. We saw that for $H \subset G$

$S(\rho_H - 1)$ has a cellular structure related to the barycentric subdivision of the standard $(|H| - 1)$ -simplex.

This leads to a cellular chain complex for computing $\underline{H}^*(S_G^{-1}; \underline{M})$ of the form

$$\underline{M}(G/G) \rightarrow \underline{M}(S_1) \rightarrow \underline{M}(S_2) \rightarrow \dots$$

where each S_i is a certain G_i -set
with $S_i^G = \emptyset$, i.e.

S_1 is a union of G/H 's for proper
subgroups H , with each such H
occurring at least once. It follows

$$\begin{aligned} \text{that } [S_1^{G_i^{-1}}, \underline{HM}] &= H^0 \text{ of the cochain } \alpha \\ &= \ker \underline{M}(G/G) \rightarrow \underline{M}(S_1) \\ &= \bigcap_{H \subset G} \ker \text{Res}_H^G: \underline{M}(G/G) \rightarrow \underline{M}(G/H) \end{aligned}$$

Can make similar stmt with

$$S_1^{G_i^{-1}} \text{ replaced by } G_i \underset{K}{\wedge} S_1^{P_{K^{-1}}}$$

for $K \in G_i$. Need to show this
holds for any positive dimensional
slice cell

$$W = G_{\mathbb{Z}} \sum_K S^{P_K n - \varepsilon} \quad \text{for } n > 0, \\ \varepsilon = 0, \downarrow \\ \text{and } K \in G. \quad (\text{for } K=e, \varepsilon=0)$$

We need a cell structure on
 $S(-nP_K - \varepsilon)$.

There is one leading to a cochain
 cx of the form (assume $K \neq e$)

$$\underline{M}(G_1/K_1) \rightarrow \underline{M}(S_1) \rightarrow \underline{M}(S_2) \rightarrow \dots$$

where S_i is a G -set with $S_i^G = \emptyset$.

We get a similar description
 of H^0 . It vanishes if \underline{M} has
 injective restriction maps. (QED)

Recall for $G = C_2$ we know

$$\underline{H}_*(S^0) = \underline{\Pi}_* \underline{H}\mathbb{Z} \quad \text{because}$$

we know $\underline{H}_* S^{n\sigma}$ for all $n \in \mathbb{Z}$.

The latter is sufficient because

$$RO(G) = \{ m + n\sigma : m, n \in \mathbb{Z} \}$$

$$\underline{H}_m S^{n\sigma} = \underline{\Pi}_{m-n\sigma} \underline{H}\mathbb{Z}$$

Now let $G = C_4$. Let λ be action of G
on \mathbb{R}^2 by $\gamma \mapsto \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\sigma: \gamma \mapsto [1]$

$$RO(G) = \{ l + m\sigma + n\lambda : l, m, n \in \mathbb{Z} \}$$

Will give method to find

$$\underline{\Pi}_{l+m\sigma+n\lambda} \underline{H}\mathbb{Z} \quad \text{for } m, n \geq 0$$

① Let $n = 0$. In the rep $l + m\sigma$,

G_1 is acting thru its quotient

$$G_1/G_1' = C_2 \quad \text{where } G_1' = \text{any subgroup of index 2.}$$

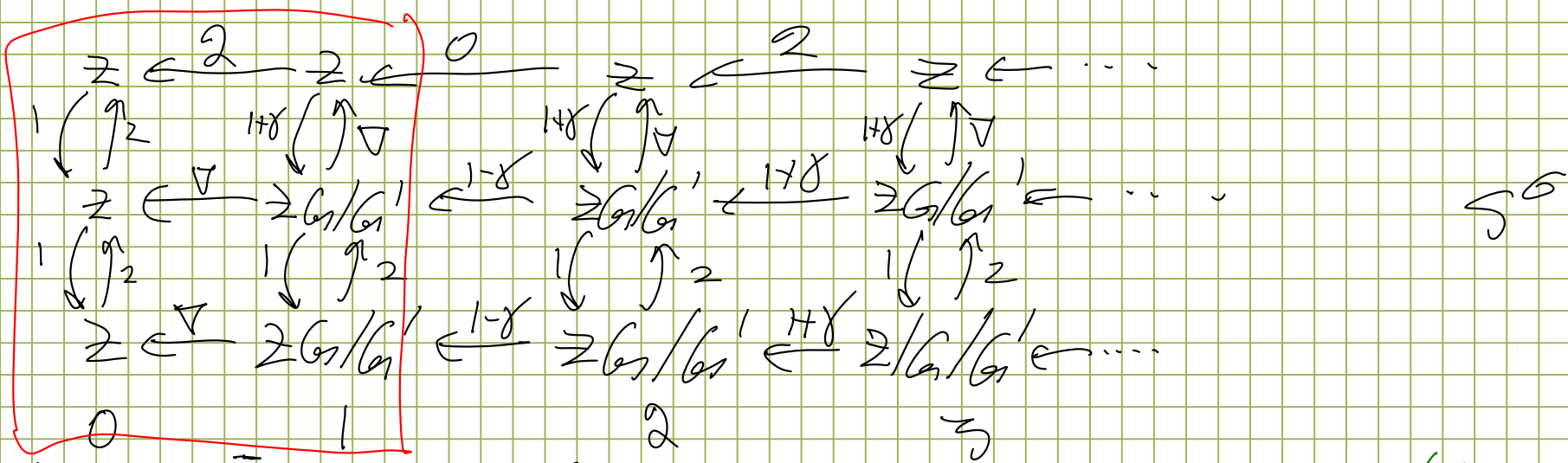
Our previous work for C_2 applies here, e.g.

$$\underline{H}_* \cong S^{m\mathbb{Z}} \quad \text{for } m > 0, \text{ we}$$

get a cellular chain complex of $\mathbb{Z}G_1$ -modules.

$$\begin{array}{ccccccc} \mathbb{Z} & \xleftarrow{\Delta} & \mathbb{Z}G_1/G_1' & \xleftarrow{1-\gamma} & \mathbb{Z}G_1/G_1' & \xleftarrow{1+\gamma} & \mathbb{Z}G_1/G_1' \xleftarrow{\dots} \\ 0 & & 1 & & 2 & & 3 \end{array}$$

The corresponding complex of Mackey functors is

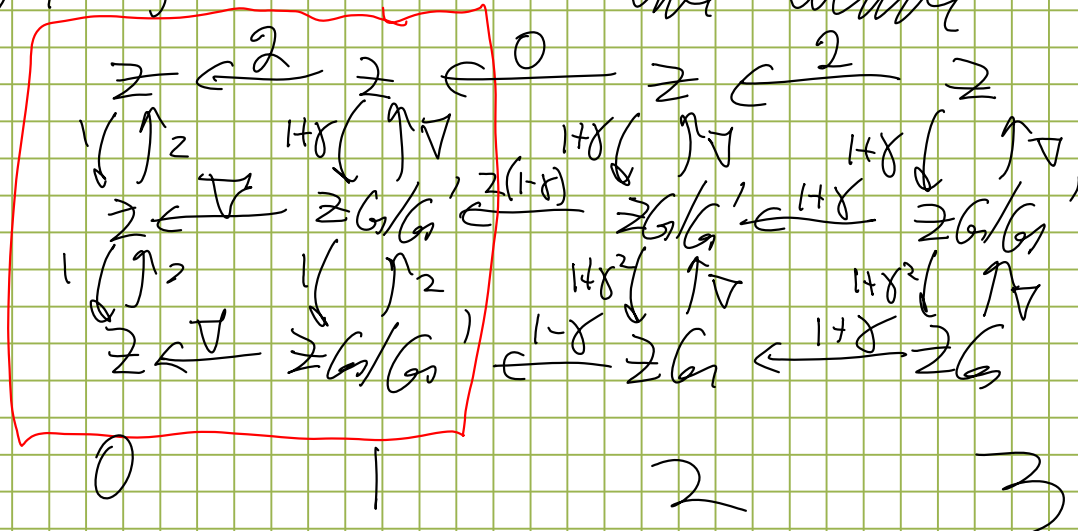


For $S^{\bar{P}_4} = 6 + 7$ we have

$X = S(6)$ $W = S(6, 7)$
 $Y = S(7)$

S^{6+7}

S^{6+7} is obtained from S^6 by attaching free G_1 -cells.



Lemma For a finite gp G , suppose

$$V = V_1 + V_2 \quad d = |V| \quad d_i = |V_i|$$

$$X = S(V_1) = S^{d_1-1}$$

$$Y = S(V_2) = S^{d_2-1}$$

$$W = S(V) = X * Y = S^{d-1}$$

Suppose X and Y have G -CW structures and the one on Y is free, i.e. G acts freely on Y . This means $V_2^H = 0$ for $H \subset G$.

Then W can be obtained from X by attaching free G -cells as follows.

$$W^{d_1-1} = X \quad \text{and} \quad W^{d_1+k} = X * Y^k \quad \text{for} \\ 0 \leq k \leq d_2-1.$$

Proof We have inclusion maps

$$X \hookrightarrow X * Y^0 \hookrightarrow X * Y^1 \hookrightarrow \dots$$

Y_0 is a free G -set. The first map is \dots \square