Note Title 10/12/2012

<u>Thm</u> A 0-slice is an Eilenberg-Mac Lane spectrum $H\underline{M}$ where $\underline{M}$ is a Mackey functor with injective restriction maps.

<u>Proof</u> We know this condition on $\underline{M}$ is necessary. We saw that for $H \subset G$

$S(\rho_H - 1)$ has a cellular structure - related to the barycentric subdivision of the standard $(|H|-1)$-simplex

This leads to a cellular chain cx for computing $\underline{H}^*(S^{\rho_G - 1}; \underline{M})$ of the form

$$\underline{M}(G/G) \to \underline{M}(S_1) \to \underline{M}(S_2) \to \cdots$$

where each S_1 is a certain G-set with $S_1^G = \emptyset$, i.e.

S_1 is a union of G/H's for proper subgps H, with each such H occurring at least once. It follows that $[S^{\rho_G - 1}, H\underline{M}] = H^0$ of the cochain cx

$$= \ker \underline{M}(G/G) \longrightarrow \underline{M}(S_1)$$

$$= \bigcap_{H \subset G} \ker \mathrm{Res}_H^G : \underline{M}(G/G) \to \underline{M}(G/H)$$

Can make similar stmt with $S^{\rho_G - 1}$ replaced by $G_+ \wedge_K S^{\rho_K - 1}$ for $K \subseteq G$. Need to show this holds for any positive dimensional slice cell

$$W = G_+ \underset{K}{\wedge} S^{\rho_K n - \varepsilon} \quad \text{for } n > 0,\ \varepsilon = 0, 1$$

and $K \subseteq G$. (For $K = e$, $\varepsilon = 0$)

We need a cell structure on $S(-n\rho_K - \varepsilon)$.

There is one leading to a cochain cx of the form (assume $K \neq e$)

$$\underline{M}(G/G) \longrightarrow \underline{M}(S_1) \longrightarrow \underline{M}(S_2) \longrightarrow \cdots$$

where S_i is a G-set with $S_i^G = \phi$.

We get a similar description of H^0. It vanishes if $\underline{M}$ has injective restriction maps. QED

Recall for $G = C_2$ we know

$$\underline{H}_\star(S^0) = \underline{\pi}_\star H\underline{\mathbb{Z}} \quad \text{because}$$

we know $\underline{H}_* S^{n\sigma}$ for all $n \in \mathbb{Z}$.

The latter is sufficient because

$$RO(G) = \{m + n\sigma : m, n \in \mathbb{Z}\}$$

$$\underline{H}_m S^{n\sigma} = \underline{\pi}_{m-n\sigma} H\underline{\mathbb{Z}}$$

Now let $G = C_4$. Let λ be action of G on $\mathbb{R}^2$ by $\gamma \mapsto \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\quad \sigma : \gamma \mapsto [-1]$

$$RO(G) = \{\ell + m\sigma + n\lambda : \ell, m, n \in \mathbb{Z}\}$$

Will give method to find

$$\underline{\pi}_{\ell + m\sigma + n\lambda} H\underline{\mathbb{Z}} \quad \text{for } mn \geq 0$$

(1) Let $n = 0$. In the rep $\ell + m\sigma$,

G is acting thru its quotient
$G/G' = C_2$ where G' = subgp of index 2.

Our previous work for C_2 applies here, e.g.
$\underline{H}_* S^{mG}$ for $m > 0$, we get a cellular chain complex of $\mathbb{Z}G$-modules.

$$\mathbb{Z} \xleftarrow{\nabla} \mathbb{Z}G/G' \xleftarrow{1-\gamma} \mathbb{Z}G/G' \xleftarrow{1+\gamma} \mathbb{Z}G/G' \longleftarrow \cdots$$
$$0 \qquad\qquad 1 \qquad\qquad 2 \qquad\qquad 3$$

The corresponding complex of Mackey functors is

$$\begin{tikzcd}[column sep=large]
\mathbb{Z} \arrow[d, bend right, "1"'] & \mathbb{Z} \arrow[l, "2"'] \arrow[d, bend right, "1+\gamma"'] & \mathbb{Z} \arrow[l, "0"'] \arrow[d, bend right, "1+\gamma"'] & \mathbb{Z} \arrow[l, "2"'] \arrow[d, bend right, "1+\gamma"'] & \cdots \arrow[l] \\
\mathbb{Z} \arrow[u, bend right, "\eta_2"'] \arrow[d, bend right, "1"'] & \mathbb{Z}G/G' \arrow[l, "\nabla"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1"'] & \mathbb{Z}G/G' \arrow[l, "1-\gamma"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1"'] & \mathbb{Z}G/G' \arrow[l, "1+\gamma"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1"'] & \cdots \arrow[l] \\
\mathbb{Z} \arrow[u, bend right, "\eta_2"'] & \mathbb{Z}G/G' \arrow[l, "\nabla"'] \arrow[u, bend right, "\eta_2"'] & \mathbb{Z}G/G' \arrow[l, "1-\gamma"'] \arrow[u, bend right, "2"'] & \mathbb{Z}G/G' \arrow[l, "1+\gamma"'] \arrow[u, bend right, "2"'] & \cdots \arrow[l] \\
0 & 1 & 2 & 3 &
\end{tikzcd}$$

S^{σ}

For $S^{\overline{\rho}_4} = \sigma + \lambda$ we have

$X = S(\sigma)$ $W = S(\sigma+\lambda)$

$Y = S(\lambda)$

$$\begin{tikzcd}[column sep=large]
\mathbb{Z} \arrow[d, bend right, "1"'] & \mathbb{Z} \arrow[l, "2"'] \arrow[d, bend right, "1+\gamma"'] & \mathbb{Z} \arrow[l, "0"'] \arrow[d, bend right, "1+\gamma"'] & \mathbb{Z} \arrow[l, "2"'] \arrow[d, bend right, "1+\gamma"'] \\
\mathbb{Z} \arrow[u, bend right, "\eta_2"'] \arrow[d, bend right, "1"'] & \mathbb{Z}G/G' \arrow[l, "\nabla"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1"'] & \mathbb{Z}G/G' \arrow[l, "2(1-\gamma)"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1+\gamma^2"'] & \mathbb{Z}G/G' \arrow[l, "1+\gamma"'] \arrow[u, bend right, "\nabla"'] \arrow[d, bend right, "1+\gamma^2"'] \\
\mathbb{Z} \arrow[u, bend right, "\eta_2"'] & \mathbb{Z}G/G' \arrow[l, "\nabla"'] \arrow[u, bend right, "\eta_2"'] & \mathbb{Z}G \arrow[l, "1-\gamma"'] \arrow[u, bend right, "\nabla"'] & \mathbb{Z}G \arrow[l, "1+\gamma"'] \arrow[u, bend right, "\nabla"'] \\
0 & 1 & 2 & 3
\end{tikzcd}$$

$S^{\sigma+\lambda}$

$S^{\sigma+\lambda}$ is obtained from S^{σ} by attaching free G-cells.

Lemma For a finite gp G, suppose
$V = V_1 + V_2 \quad d = |V| \quad d_i = |V_i|$

$$X = S(V_1) = S^{d_1 - 1}$$

$$Y = S(V_2) = S^{d_2 - 1}$$

$$W = S(V) = X * Y = S^{d-1}$$

Suppose X and Y have G-CW structures and the one on Y is free, i.e. G acts freely on Y. This means $V_2^H = 0$ for $H \subset G$. Then W can be obtained from X by attaching free G-cells as follows:

$W^{d_1-1} = X$ and $W^{d_1+k} = X * Y^k$ for $0 \leq k \leq d_2 - 1$.

Proof We have inclusion maps

$$X \hookrightarrow X * Y^0 \hookrightarrow X * Y^1 \hookrightarrow \cdots$$

Y_0 is a free G-set. The first map is ..... .