Lemma: \( G \) is a finite group
\[ V = V_1 + V_2 \]
reps of \( G \)
\[ d_i = \dim V_i \quad i = 1, 2 \]
\[ X = S(V_1) = S^{d_1-1} \]
\[ Y = S(V_2) = S^{d_2-1} \]
\[ W = X \times Y = S(V) = S^{d_1 + d_2 - 1} \]
\[ V_2 = 0 \quad \text{for any nontrivial } H \leq G \]
so \( Y \) is a free \( G \)-space
\( X \) and \( Y \) have \( G \)-CW structure when the one on \( Y \) is free.
Then $W$ can be obtained from $X$ by attaching free $G$-cells as follows. Let $Y^0 < Y^1 < Y^2 < \cdots < Y^n$ be the skeletons of $Y$. Then

$W_{n-1} \hookrightarrow W_n \hookrightarrow \cdots \hookrightarrow W_{n+d_2}$

All but the first inclusion are functorial on the maps $Y^i \hookrightarrow Y^{i+1}$.

Recall $X \times Y = X \times I \times Y / \{ (x,0,y) \sim (x,0,y'), (x,1,y) \sim (x',1,y) \}$.

Picture from $Y_0 = S^3 = C_3$. 

\[ X \ast C_3 = \text{triple cone on } X \]
\[ G \text{-action on } X \text{ and } C_3 \]
\[ \text{induce one on } X \ast C_3 \]
\[ i_0 : X \rightarrow X \ast C_3 \text{ inclusion} \]

When \( X = S^d \cup 1 \), we are attaching three \( d \)-cells to get \( X \ast C_3 \).

Similar thing happens at each stage and we have a \( G \)-CW structure on \( W \).

\( \text{QED} \).

Back to \( C_4 \). Want to find \( H_* S^m \mathbb{C}G + n \lambda \).

We have a \( G \)-CW structure on \( H_* S^m \mathbb{C}G + n \lambda \) for \( m, n \geq 0 \).
and want to extend it to one on $G_0 + n \gamma$. The Lemma applies because $\gamma$ is a fixed rep of $G_0 = G$, let $G' \subseteq G$.

\[ e.g., \quad m = 2, \quad n = 2 \]

\[ 2 \subseteq 2G/G, \quad e^{-\gamma} \quad 2G/G \leq 2G \leq 2G' \leq 2G' \leq 2G \leq 2G \]

\[ H \quad (5^2 G + 2A) \quad \text{underlain by} \quad H \quad \gamma \]

\[ 2G' = 2 + 2G + 2A \]

\[ 2G/G' = 2 \ominus 1 \quad 8 \frac{1}{\gamma} \quad (2G/G')^{\frac{1}{2}} = \ominus \frac{3}{2} + \delta \frac{1}{\gamma} \]

\[ \text{index} 2 \]

\[ \text{under} \gamma \]
\[(2G_1)^2 = Z_31 + Z^2, \quad Z + Z^3 \]  
\[(2G_2)^2 = Z_31 + Z + Z^2 + Z^3 \]