Note Title 10/16/2012

Mackey functor chain cx for $\underline{H}_*\, S^{-\rho_4}$

$$
\begin{tikzcd}
\mathbb{Z} \arrow[r, "1"] \arrow[d, shift right, "1"'] & \mathbb{Z} \arrow[r, "0"] \arrow[d, shift right] & \mathbb{Z} \arrow[r, "2"] \arrow[d, shift right, "\Delta"'] & \mathbb{Z} \arrow[d, shift right, "\Delta"'] \\
\mathbb{Z} \arrow[r, "\Delta"] \arrow[u, shift right] \arrow[d, shift right, "1"'] & \mathbb{Z}[G/G'] \arrow[r, "1-\gamma"] \arrow[u, shift right, "\nabla"'] \arrow[d, shift right, "1"'] & \mathbb{Z}[G/G'] \arrow[r, "1+\gamma"] \arrow[u, shift right, "\nabla"'] \arrow[d, shift right, "1+\gamma^2"'] & \mathbb{Z}[G/G'] \arrow[u, shift right, "\nabla"'] \arrow[d, shift right, "1+\gamma^2"'] \\
\mathbb{Z} \arrow[r, "1+\gamma"] \arrow[u, shift right, "2"'] & \mathbb{Z}[G/G'] \arrow[r, "\theta"] \arrow[u, shift right, "2"'] & \mathbb{Z}[G] \arrow[r, "1+\gamma"] \arrow[u, shift right, "\nabla"'] & \mathbb{Z}[G] \arrow[u, shift right, "\nabla"']
\end{tikzcd}
$$

$= C_{-1}$, $= C_{-2}$, $= C_{-3}$, $= C_{-4}$

$\theta = (1-\gamma)(1+\gamma^2)$

$\underline{H}_*$: $\mathbb{Z}/2$; 0; $\underline{\mathbb{Z}}_-$; $\underline{\mathbb{Z}}_-$; $\square$

$$C_{-3}^{G'} = C_{-4}^{G'} = \mathbb{Z}\{1+\gamma^2,\ \gamma+\gamma^3\} \qquad C_{-3}^{G} = C_{-4}^{G} = \mathbb{Z}\{1+\gamma+\gamma^2+\gamma^3\}$$

$$C_{-2} = \mathbb{Z}G/G' = \mathbb{Z}\{1,\ \gamma\} \qquad C_{-2}^{G} = \mathbb{Z}\{1+\gamma\}$$

Note the chart shows

$$\pi_{i+j\rho_4} H\underline{\mathbb{Z}} = \pi_{i+j(\sigma+\lambda)} H\underline{\mathbb{Z}} \quad \text{for all } i, j$$

It does not show $\pi_{i+j\sigma+k\lambda} H\underline{\mathbb{Z}}$ for $j \neq k$.

Answer for $jk < 0$ is unknown.

Consider other small gps.

$G = S_3 =$ symm gp on 3 letters

$$\begin{tikzcd}
& M(G/G) \arrow[dl, bend right] \arrow[dr, bend left] & \\
M(G/C_2) \arrow[ur, bend left] \arrow[dr, bend right] & & M(G/C_3) \arrow[ul, bend right] \arrow[dl, bend left] \\
& \underline{M}(G/e) \arrow[ul, bend right] \arrow[ur, bend right] &
\end{tikzcd}$$

$G = Q_8$ = quaternion gp

3 nonconjugate subgps $\cong C_4$ $\quad H_1, H_2, H_3$

1 subgp of order 2 $\quad C$

1 trivial subgp

$$\begin{tikzcd}
 & M(G/G) \arrow[dl, shift right] \arrow[d, shift right] \arrow[dr, shift right] & & \\
M(G/H_1) \arrow[ur, shift right] \arrow[dr, shift right] & M(G/H_2) \arrow[u, shift right] \arrow[d, shift right] & M(G/H_3) \arrow[ul, shift right] \arrow[dl, shift right] & \{M(G/H_i)\} \\
 & M(G/C) \arrow[ul, shift right] \arrow[u, shift right] \arrow[ur, shift right] \arrow[d, shift right] & & \\
 & M(G/e) \arrow[u, shift right] & &
\end{tikzcd}$$

$G \cong C_2 \times C_2 = Q_8 / \bar{C}$

$\exists$ nonconjugate subgps of index (and order) 2