How to construct an Eilenberg-Mac Lane spectrum $HM$ for a Mackey functor $M$.

An $RO(G)$ graded cohomology theory is a set functor $F_\ast : G$-spectra \( \rightarrow RO(G)$-graded algebras of $G$-spectra satisfying the Eilenberg-Steenrod axioms.

An (Brown) $G$-functor $k$ as above is representable by a $G$-spectrum $k(x) = [X,k]$ if $k$ satisfies weak axiom.
and MUS.

Any $E_0$ as above is represented by a $G$-spectrum

Any $G$-graded theory $F$ on $G$-spectrum

is represented by a $G$-spectrum $F'$ and it extends to an ROC($G$) - graded theory by $F'(X) = F_0 (Σ^n X)$

Thus given a Mackey functor $M$ there

in a spectrum $HM$ with

$\text{HM} = \left\{ \begin{array}{ll}
M & i = 0 \\
0 & i \neq 0
\end{array} \right.$ for $i \in \mathbb{Z}$
If we produce a ℤ-graded theory:

\( X \) is a \( G \)-CW spectrum

\( X^n = n \)-skeleton

Define a chain \( CX \) of Mackey functors by

\[
C_\eta^n(X) = \prod_m (X^n / X^{n-1})
\]

with boundary operator \( d \).

Note \( X^n / X^{n-1} \) = wedge of \( n \)-sphere \( G \)-skeleton

\( S^n_\eta = G/H^n \wedge S^n \)

We dualize this chain \( CX \)

\[
C_G^n (X; M) = \text{Hom}(C_\eta^n(X), M)
\]

with coboundary

This is a cochain \( CX \) of abelian \( \mathbb{G}_k \).

Call its cohom \( H^n_G (X; M) \).
It is easy to check that this satisfies the axioms required by Brown. We get a dimension axiom as follows:

\[ S^0_H = \mathcal{G}_0 \cap \mathcal{S}^0 = \mathcal{G}_0 / H^+ \]

\[ C_i (\mathcal{G}_0 / H^+) = 0 \text{ for } i \neq 0 \]

\[ C_0 (\mathcal{G}_0 / H^+) = \prod_0 (\mathcal{G}_0 / H^+) \]

\[ \prod_0 (\mathcal{G}_0 / H^+) \cdot (\mathcal{G}_0 / K) = \prod_0 (\mathcal{G}_0 / H)^K \]

\[ = \sum \begin{cases} 0 & \text{if } K \not\subset H \\ 2G / H & \text{if } K \subset H \end{cases} \]

\[ C^i (\mathcal{G}_0 / H^+, M) = \{ \text{Hom}(C_0 (\mathcal{G}_0 / H^+), M) \} \text{ if } i = 0 \]
\[ \begin{array}{cl}
\{ 0 \} & i \neq 0 \\
\mathbb{I} M (G/H) & i = 0 
\end{array} \]

Hence we have \( H_G^* (\mathbb{I}_M, M) = \mathbb{I} M (G/H) \) \( (\star) \)

This theory is representable a \( G \)-coalgebra \( H M \) with \( \mathbb{I}_M \) concentrated in \( \text{dim } D \).

\( (\star) \Rightarrow \quad \mathbb{I}_M H M = \left\{ \begin{array}{cl}
0 & i = 0 \\
\mathbb{I}_M & i \neq 0
\end{array} \right. \)

Since \( \mathbb{I}_M H M (G/H) = \left[ G/H, H M \right]_G^0 \)

\[ = H_G^0 (G/H, M) = \mathbb{I} M (G/H) \]

by previous computation (QED)