Note Title 10/2/2012

Prop 2.22 Let $H \subset G$, $h = |H|$, $g = |G|$
For $n \le -1$,

$$G_+ \wedge_H S^n \in \mathcal{A}_{\ge (n+1)h - 1}$$
$$\notin \mathcal{A}_{\ge (n+1)h}$$

Earlier we saw that for $n \ge -1$

$$G_+ \wedge S^n \in \mathcal{A}_{\ge n}$$
$$\notin \mathcal{A}_{\ge n+1}$$

Example $G = C_2 \supseteq H$
For $n \le -1$, $S^n \in \mathcal{A}_{\ge 2n+1}$ but $S^n \notin S_{\ge 2n+2}$

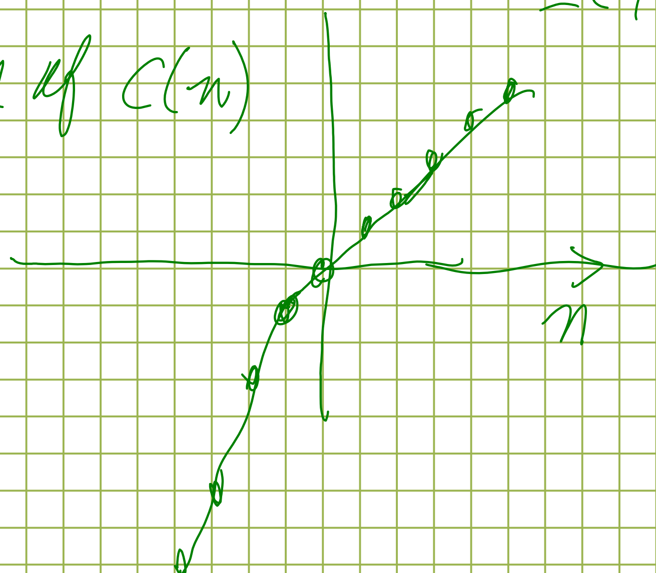
$n \geq -1$ $\quad S^n \in A_{\geq n}$ but $S^n \notin A_{\geq n+1}$

$S^{-2} \in A_{\geq -3}$

$S^{-3} \in A_{\geq -5}$

Suppose $S^n \in A_{\geq c(n)}$ but $S^n \notin A_{\geq 1+c(n)}$

graph of $c(n)$

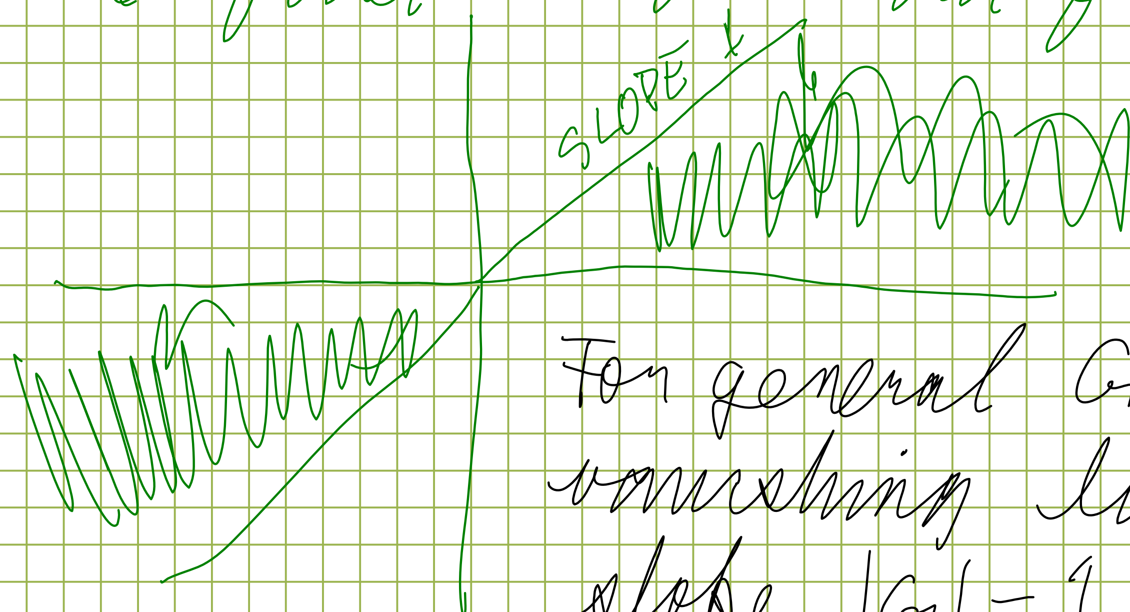


We can get a convergence theorem for the slice SS as follows:

All n-slice cells are

$$\begin{cases} (\lceil n/|G| \rceil - 1)\text{-connected} & \text{for } n \geq 0 \\ (n-1)\text{-connected} & \text{for } n < 0 \end{cases}$$

Let $G = C_2$. Slice 'SS is confined to the first and third quadrants



For general G the vanishing line has slope $|G| - 1$.

Note: For $n \geq 0$, $S^{n\rho_G}$ has bottom cell in dim n, top cell in dim $n|G|$

For $n < 0$, $S^{n\rho_G}$ has bottom cell in dim $n|G|$, top cell in dim n.

Multiplicative properties of slice filtration:

In general if $X \in \mathcal{S}_{\geq m}$ and $Y \in \mathcal{S}_{\geq n}$, it is <u>not</u> true that $X \wedge Y \in \mathcal{S}_{\geq m+n}$

e.g. $G = C_2$, $X = Y = S^{-1}$

Prop 2.23 If $X \geq 0$ and $Y \geq n$ then $X \wedge Y \geq n$.

Proof: We saw before that $\mathcal{S}_{\geq 0}$ is generated by $\{G/H_+ : H \subset G\}$ and $G/H_+ \wedge \mathcal{S}_{\geq n} \subset \mathcal{S}_{\geq n}$. QED

Cor 2.24 If $X \geq k|G|$ and $Y \geq n$ then $X \wedge Y \geq k|G| + n$.

Proof We $S^{-k\rho_G} \wedge X \geq 0$

so $S^{-k\rho_G} \wedge X \wedge Y \in \mathcal{S}_{\geq n}$

Smash with $S^{k\rho_G}$ and get

$X \wedge Y \in S^{k\rho_G} \wedge \mathcal{S}_{\geq n} = \mathcal{S}_{n+k|G|}$ QED

Prop 3.1 If $Y \in \mathcal{A}_{\geq n}$ then $\Sigma Y \in \mathcal{A}_{\geq 1+n}$.

but $\Sigma^{-1} Y \notin \mathcal{A}_{\geq n-1}$

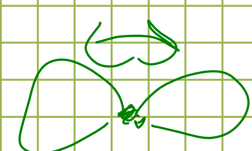
__Proof__ Argue by induction on $|G|$.
Statement is true for $|G| = 1$.
Consider the cofiber sequence

$$X = \Sigma\left(S(\rho_G - 1)_+\right) \longrightarrow S^1 \longrightarrow S^{\rho_G}$$

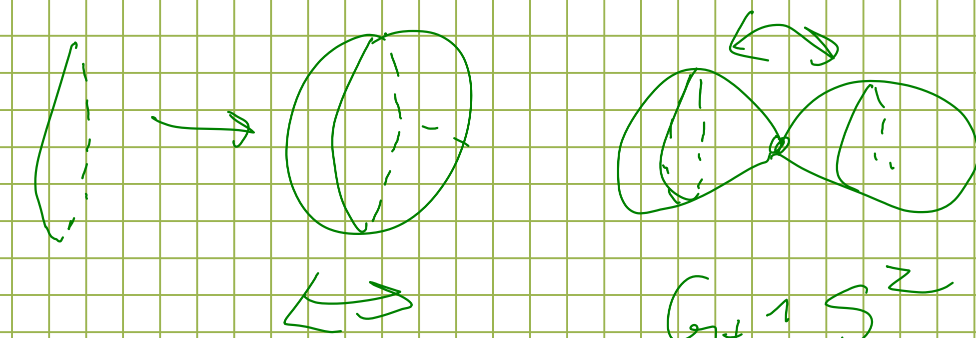
$\rho_G - 1 = \bar{\rho}_G$ = reduced regular rep.

e.g. $G = C_2$

$$S^1 \longrightarrow S^{\rho_2}$$

$\Sigma S(\rho_G - 1)_+ = \Sigma G_+ = G_+ \wedge S^1 =$ 

$G_+ \wedge S^1 \longrightarrow$



$G_+ \wedge S^2$

X is 0-connected and G acts s.t. $X^G = pt.$ This means X is built from $G/H_+ \wedge S^k$ with $k \geq 1$ and $H \neq G$. By induction we have $X \in \mathcal{A}_{\geq 1}$

$$\underset{\geq 1}{X} \longrightarrow S^1 \longrightarrow \underset{\geq |G|}{S^\rho}$$

Smash with Y

$$Y \in S_{\geq n}$$

$$\underset{\geq n+1}{X \wedge Y} \longrightarrow \underset{\geq 1+n}{\Sigma Y} \longrightarrow \underset{\geq n+g}{Y \wedge S^p}$$

$$Y \wedge S^p \in \mathcal{A}_{\geq n+g}$$

$$X \wedge Y \in \mathcal{A}_{\geq 1+n}$$

QED.