Note Title 10/30/2012

Want to discuss slice SS for ku and K. $G = C_2$. There is a map $ku \to K$ of G-spectra.

ku could be called $k\mathbb{R}$ or $k\mathbb{R}u$

Properties of ku and K

as ordinary spectra

1) $\pi_i\, ku = \begin{cases} \mathbb{Z} & \text{for } i \geq 0 \text{ even} \\ 0 & \text{else} \end{cases}$

$\pi_i\, K = \begin{cases} \mathbb{Z} & \text{for } i \text{ even} \\ 0 & i \text{ odd} \end{cases}$

These are ring spectra

Let $x \in \pi_2\, ku \xrightarrow{\cong} \pi_2 K$ be a generator

Then $\pi_* ku = \mathbb{Z}[x]$ and $\pi_* K = \mathbb{Z}[x, x^{-1}]$

K can be obtained from ku as follows:

$$S^2 \xrightarrow{x} ku$$

$$\Sigma^2 ku \xrightarrow{x \wedge ku} ku \wedge ku \xrightarrow{m} ku$$

$$ku \longrightarrow \Sigma^{-2} ku \longrightarrow \Sigma^{-4} ku \longrightarrow \cdots\cdots$$

$$\varinjlim \Sigma^{-2i} ku = K \quad \text{and} \quad \pi_* K = x^{-1} \pi_* ku$$

2) ku and K as G-spectra:

x corresponds to an equiv map

$$S^\rho \xrightarrow{x} ku \longrightarrow K \qquad \rho = 1 + \sigma = \text{regular rep of } G$$

$$x \in \underline{\pi}_\rho \, bu(G/G) \longrightarrow \underline{\pi}_\rho K(G/G)$$

For a generator γ of G,

$$Res_1^2(x) \in \underline{\pi}_\rho \, bu(G/e) \simeq \underline{\pi}_2 \, bu(G/e)$$

$$\gamma(Res_1^2(x)) = -Res_1^2(x)$$

3) (Deep Thm) Slices of bu and K

$$P_i^i \, bu = \begin{cases} * & \text{for } i \text{ odd and for } i < 0 \\ S^{i\rho/2} \wedge H\underline{\mathbb{Z}} & \text{for } i \geq 0 \text{ even.} \end{cases}$$

$$P_i^i \, K = \begin{cases} * & \text{for } i \text{ odd} \\ S^{i\rho/2} \wedge H\underline{\mathbb{Z}} & \text{for } i \text{ even} \end{cases}$$

We know $\underline{\pi}_\bigstar H\underline{\mathbb{Z}}$, so we know the E_2-term of the slice SS.

Recall we have elements

$$a_\sigma \in \underline{\pi}_0 S^\sigma(G/G) = \underline{\pi}_{-\sigma} S^0(G/G)$$

Since ku and K are modules over S^0, we get an element in

$$a_\sigma \in E_2^{1,1-\sigma} X(G/G) \quad \text{for } X = ku \text{ or } K$$

recall $E_2^{s,t}$ converges to a subquotient of $\underline{\pi}_{t-s}$ where $s \in \mathbb{Z}$, $t \in RO(G)$.

In the chart, $\underline{E}_2^{s,t}$ is shown at the point $(t-s, s)$

$$d_r : \underline{E}_r^{s,t} \longrightarrow \underline{E}_r^{s+r, t+r-1}$$

$$u_\sigma \in \pi_{1-\sigma} H\underline{\mathbb{Z}}(G/e)$$

$$u_{2\sigma} \in \pi_{2-2\sigma} H\underline{\mathbb{Z}}(G/G) \qquad \mathrm{Res}_1^2 u_{2\sigma} = u_\sigma^2$$

$\chi \in \underline{\pi}_{1+\sigma} \mathrm{lm}(G/G)$ and $\underline{E}_2^{0,1+\sigma}(\mathrm{lm})(G/G)$

and similarly for K

$$a = a_\sigma \in \underline{E}_2^{1,1-\sigma} \mathrm{lm}(G/G)$$

$$\chi a \in \underline{E}_2^{1,2} \mathrm{lm}(G/G)$$

$(xa)^1 \in \underline{E}_2^{i,2i}(G/G)$ for $i \geq 0$

$u_G \mathrm{Res}_1^2(x) \in \underline{E}_2^{0,2}(G/G)$ This gp is $\mathbb{Z}$ generated by this element.

$e_{nG} \in \pi_{nG-n} H\underline{\mathbb{Z}}(G/e)$ for $n \geq 1$

has infinite order for even n

order 2 for odd n

The elements x, a, are permanent cycles, but the u's are not.

$e_{nG} \mathrm{Res}_1^2(x^{-n}) \in \underline{E}_2^{0,-2n}(G/e)$

is a permanent cycle because there are no possible targets for a differential.

$u_{2\sigma} \chi^2 \in E_2^{0,4}(G/G)$ $\qquad u_{2\sigma} \in \pi_{-2-2\sigma} H\underline{\mathbb{Z}}(G/G)$

$u_\sigma \, Res_1^2(\chi) \in E_2^{0,2}(G/e)$ $\qquad u_\sigma \in \pi_{-1-\sigma} H\underline{\mathbb{Z}}(G/e)$

Thm (Slice differentials)

$$d_3(u_{2\sigma}\chi^2) = a_\sigma^3 \chi^3$$

$$E_3^{0,4}(G/G) \xrightarrow{d_3} E_3^{3,6}(G/G)$$

Since χ is an invertible permanent cycle.

$u_{2\sigma}\chi^{-1} \longrightarrow a_\sigma^3$

so $a_\sigma^3 = 0$ in $E_4(G/G)$

$E_4^{s,t}(G/G) = 0$ for $s \geq 3$

$u_{4\sigma}\chi^4 \in E_4^{0,8}(G/G)$ is a permanent cycle.