COMPUTATION PRECEDES THEORY

Recall: $G = C_2$. There is a G-spectrum K (complex K-theory where G acts via conjugation) with slices

DEEP THEOREM (DUGGER)

$$P_i^i K = \begin{cases} * & \text{for } i \text{ odd} \\ S^{i\rho/2} \wedge H\underline{\mathbb{Z}} & \text{for } i \text{ even} \end{cases}$$

We know the $\underline{E}_2$-term because we know $\underline{\pi}_* H\underline{\mathbb{Z}}$.

Slice differentials theorem $\quad d_3(u_{2\sigma} x^2) = a^3 x^3$

where $a = a_6 \in E_2^{1,1-6}(G/G)$

$x \in E_2^{0,1+6}(G/G)$

$u_{26} \in E_2^{0,2-26}(G/G)$

hence $x^2 u_{26} \in E_2^{0,4}(G/G)$

$a^3 x^3 \in E_2^{3,6}(G/G)$.

Consequences:

1) Since x is an invertible permanent cycle, $d_3(x^{-1} u_{26}) = a^3$. Hence $E_4^{s,t} = 0$ for $s \geq 3$.

2) $\pi_* \Phi^G X$ = 2-graded portion of $a^{-1} \pi_\star X(G/G)$

Hence $\Phi^G K$ is contractible

3) $\Delta = \mu_{2\sigma}^2 \chi^4 \in E_2^{0,8}(G/G)$ is a permanent cycle.

Periodicity Theorem $K \simeq \Sigma^8 K$ as G-spectra, so $\underline{\pi}_n(K) \cong \underline{\pi}_{n+8}(K)$

Proof $\Delta : S^8 \longrightarrow K$, $\Delta \in \underline{\pi}_8(G/G)$

$$\mathrm{Res}_1^2(\Delta) = \mathrm{Res}_1^2(\mu_{2\sigma})^2\, \mathrm{Res}_1^2(\chi)^4$$
$$= \mathrm{Res}_1^2(\chi)^4 \in \underline{\pi}_8(G/G) = \pi_8^u K$$

It is invertible because χ is.

The map Δ leads to

$$\Sigma^{8}K \xrightarrow{\Delta \wedge K} K \wedge K \xrightarrow{m} K$$

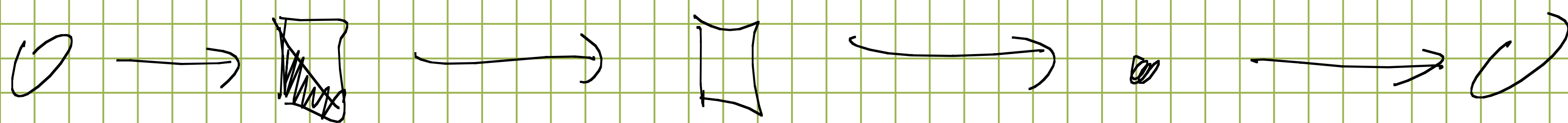
It is an ordinary ~~hty~~ equivalence. To show it is a G-equivalence it suffices to show $\tilde{\Phi}^{G}(\sim)$ is an equivalence. This holds since $\Phi^{G}K = pt.$ QED

Periodicity forces some differentials in the third quadrant as shown in the chart.

In the third quadrant we have

$$0 \to \underline{E}_4^{0,-8i} \to \underline{\Pi}_{-8} \to \underline{E}_4^{-2,-2-8i} \to 0 \qquad i>0$$

$$\underline{\Pi}_{-8} = \underline{\Pi}_0$$



$$\begin{tikzcd}
0 & \mathbb{Z} \arrow[r, "2"] \arrow[d, bend right, "2"'] & \mathbb{Z} \arrow[r] \arrow[d, bend right, "1"'] & \mathbb{Z}/2 \arrow[r] \arrow[d, bend right] & 0 \\
0 \arrow[r] & \mathbb{Z} \arrow[r, "1"] \arrow[u, bend right, "1"'] & \mathbb{Z} \arrow[r] \arrow[u, bend right, "2"'] & 0 \arrow[r] \arrow[u, bend right] & 0
\end{tikzcd}$$

This leads to an exotic restriction shown in green.

Similarly, in the first quadrant we have exotic transfers in

dimensions 2 mod 8
shown in blue.