

MATH 550

Note Title

10/5/2012

Assume G is a cyclic 2-gp

Recall we have

$$\rho_V \in \pi_{-V} \cong \mathbb{Z} (G/G_V) \quad \text{for } V^G = 0$$

$$\mu_V \in \pi_{|V|-V} \cong \mathbb{Z} (G/G_{\det V})$$

where $G_V =$ isotropy gp of V

$=$ largest subgroup of G acting trivially on V

$\det(V) =$ determinant rep associated with V .

if V is oriented then $\det(V) = 1$

$G_{\det V} = G$, otherwise

$G_{\det V}$ has index 2.

Also we have $e_V \in \prod_{V-|V|} H^2(G/G_V)$
given by the underlying equiv

$$S^V \rightarrow S^{|V|} \quad |V| = \dim V$$

Lemma 9 (in a HHR preprint)

i) $a_{V+W} = a_V a_W$ and $m_V m_W = m_{V+W}$

ii) $|G/G_V| a_V = 0$

iii) For oriented V , $T_{G_V}^G(e_V)$ has
infinite order, as does $T_{G_{V+G}}^G(e_{V+G})$

where G' is the subgroup of index 2,
which is $G_{\det \sigma} = G_6$

$\text{TM}_{G_{V+0}}^{G'}(e_{V+0})$ has order 2 if $|V| > 0$

$$\text{TM}_{G_6}^{G'}(e_6) = 0.$$

iv) For oriented V , $\mu_V \text{TM}_{G_V}^{G'}(e_V) = |G'/G_V|$

$$\text{in } \mathbb{I}_0 \text{HZ}(G'/G) = \mathbb{Z}$$

3 more similar statements.

Back to $G_1 = G_2$. We have calculated

$$\prod_{i+j \in \mathfrak{o}} H \cong \text{for } j \in \mathfrak{o}, i \in \mathbb{Z}.$$

$$M_{2\mathfrak{o}} \in \prod_{-2-2\mathfrak{o}} H \cong (G/G)$$

$$M_{\mathfrak{o}} \in \prod_{-\mathfrak{o}} H \cong (G/e)$$

$$\text{with } M_{\mathfrak{o}}^2 = \text{Res}_e^G M_{2\mathfrak{o}}$$

$$A_{\mathfrak{o}} \in \prod_{-\mathfrak{o}} H \cong (G/G)$$

$$\text{Claim } M_{2\mathfrak{o}}^i A_{\mathfrak{o}}^j = M_{2i\mathfrak{o}} A_{j\mathfrak{o}} \text{ for } i, j \geq 0$$

$$\in \prod_{-2i-(2i+j)\mathfrak{o}} H \cong (G/G)$$

$$= \begin{cases} \mathbb{Z} & \text{if } j=0 \\ \mathbb{Z}/2 & \text{if } j > 0 \end{cases}$$

In each case this element generates the gp.

$$v_{\mathbb{G}}^i \in \pi_{i-i\mathbb{G}} \cong (G/e) = \mathbb{Z}$$

is the generator.

To get at $\pi_{i+j\mathbb{G}} \cong \mathbb{Z}$ for $j > 0$ we need to find $H_*(S^{-j\mathbb{G}})$ for $j > 0$

$S^{-j\mathbb{G}}$ is the Spanier-Whitehead

dual of $S^{j\mathbb{G}}$, i.e. $S^{-j\mathbb{G}} \wedge S^{j\mathbb{G}} = S^0$

A cellular chain cx for $S^{-j\mathbb{G}}$ can

be obtained by dualizing the one

for S^{j_0} . ("dualizing" to be explained)
 $C_* S^{j_0}$ has the form

$$\begin{array}{ccccccc}
 & & 1 & & 2 & & 3 & & 4 \\
 \mathbb{Z} & \xleftarrow{\Delta} & \mathbb{Z}G & \xleftarrow{1-\gamma} & \mathbb{Z}G & \xleftarrow{1+\gamma} & \mathbb{Z}G & \xleftarrow{1-\gamma} & \mathbb{Z}G \xleftarrow{\dots}
 \end{array}$$

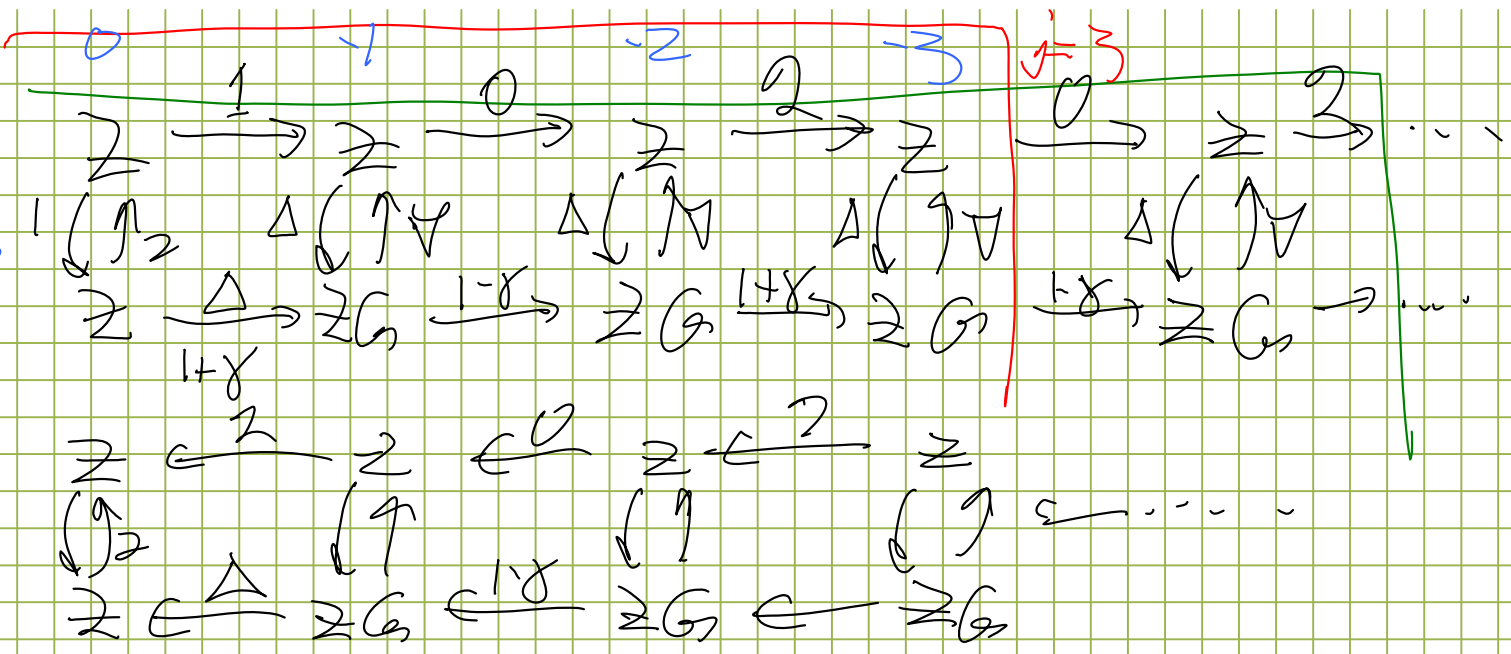
Apply $\text{Hom}_{\mathbb{Z}}(-, \mathbb{Z})$ to this and
 get a cochain C^*

$$\begin{array}{ccccccc}
 \mathbb{Z} & \xrightarrow{1+\gamma} & \mathbb{Z}G & \xrightarrow{1-\gamma} & \mathbb{Z}G & \xrightarrow{1+\gamma} & \mathbb{Z}G & \xrightarrow{1-\gamma} & \mathbb{Z}G \rightarrow \dots \\
 0 & & -1 & & -2 & & -3 & & -4
 \end{array}$$

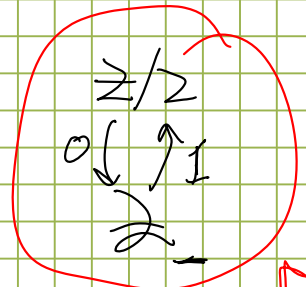
This is a cellular chain C^* for S^{j_0}

apply the fixed point Mackey functor
games

$j=2$

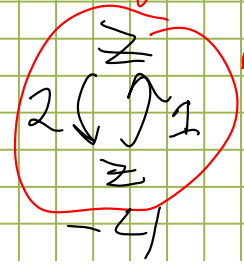


| | | | | |
|---------------|---------------|---------------|---------------|---------------|
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| $\mathbb{Z}G$ | $\mathbb{Z}G$ | $\mathbb{Z}G$ | $\mathbb{Z}G$ | $\mathbb{Z}G$ |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
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| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |
| \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} | \mathbb{Z} |



$H_x S^{-3G}$

New Mackey function



$H_x S^{-4G}$

M

$M(G_1/G_2)$

$R_{G_1}^Z$

\downarrow

\uparrow

$T_{G_1}^Z$

$M(G_1/e)$

= $\mathbb{Z}G_1$ -module