Recall

\[ H(a, t) = P(a) \mathcal{P}(t) \mathcal{F}(a, t) \in \mathcal{P} \times \mathcal{M} \times \mathcal{U} \left[ \mathbb{C}, \mathbb{R} \right] \]

where

\[ H(a, t) = \sum \left[ \mathcal{H}^m \mathcal{J}^m \mathcal{J}^n \right] \text{ where} \]

\[ H^m \mathcal{J}^n \text{ is the Milnor hypersurface in } \mathbb{C} \mathcal{P}^m \times \mathbb{C} \mathcal{P}^n \]

\[ P(a) = \sum \left[ \mathcal{C} \mathcal{P}^m \right] \mathcal{J}^m \]

\[ \mathcal{F}(a, t) = \sum a_{m, n} \mathcal{J}^m \mathcal{J}^n \]
There is an invariant of smooth projective algebraic varieties called Todd genus $td$. It is a cobordism invariant.

Disjoint unions $\rightarrow$ sums
Cartesian products $\rightarrow$ products

Can be defined in terms of Chow numbers.

It is a hom $\Sigma^U = \pi_* MV \rightarrow \mathbb{Z}$

see Hirzebruch; Topological methods in algebraic geometry.
\text{ad } (C \mathbb{P}^n) = 1. \text{ Redefine it as a graded homomorphism } \tau_x \text{ MV} \to \mathbb{Z} [u] = R

\text{where } |u| = 2

\mathbb{Z} \left[ u, u^{-1} \right] = [C \mathbb{P}^n] \mapsto u^n

What is the resulting FGL over \text{R}?

Recall the FGL logarithm

\log x = \sum_{m \geq 0} \frac{[C \mathbb{P}^n^n x^n]}{n!} \in \tau_x \text{ MV} \otimes \mathbb{Q} [F \mathbb{P}^n]

\log f(x, y) = \log(x) + \log(y)

\text{MYSHENKO 1967.}
\( \log(x) \) is an isomorphism with the additive \( \text{FGM} \) defined over \( \mathbb{R} \times \text{MV} \otimes \mathbb{Q} \).

\[
\frac{d \log(x)}{dx} = P(x) = \sum \prod \alpha \beta \gamma x^n
\]

where

\[
F_2(x, 0) = \frac{1}{F_2(x, 0)}
\]

\[
F_2(x, y) = \frac{2 F(x, y)}{\partial y}
\]

Applying the Todd genus to \( \log x \) gives
\[ \log x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \]

\[ \Rightarrow F(x, y) = x + y - \ln xy \]

If we set \( n = 1 \), we get

\[ \log x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{x^n}{n} = -\ln (1-x) \]

\[ \text{MV} \xrightarrow{t_2} R \geq \sum_{n=1}^{\infty} \text{defines the FGL2/R given above}. \]

Consider the function
$X \mapsto \text{MV}_*(X) \otimes_{\text{MV}_*} R := K_*(X)$

where $R$ is an MV$_*$-module via $td$.

This function has an exactness property needed to make it a generalized homology theory.

(This requires $R$ to be invertible.)

There is a spectrum $K$ that represents this function, i.e. $K_+ x \simeq \text{MV}_*(X_+ / K)$.

It is the same as the $K$ defined in terms of complex vector bundles.
This was known by 1965.

\[ K_*(X) = \text{MU}_*(X) \otimes \text{MU}_* \]

This all works equivariantly.

Replace \( K \) by \( K_R \).

Replace \( \text{MU} \) by \( \text{MU}_R \).

Recall we know the differentials in this SS for \( \text{MU}_R \).

There there is an equivariant map

\[ \text{MU}_{12} \rightarrow K_R \]
by the map $MV \to K$ related to the Todd genus. Hence we get a map of slice spectral sequences. The $d_3$ in the slice $SS$ for $MV$ maps to the $d_3$ in the slice $SS$ for $KR$. Recall that 

$\Phi^2 : MV_R \cong MO$ and it is MO

was determined by Thom in 1954.