Note Title 11/14/2012

Recall

$$H(s,t) = P(s)\,P(t)\,F(s,t) \in \pi_* MU[[s,t]]$$

where

$$H(s,t) = \sum [H^{m,n}]\, s^m t^n$$ where

$H^{m,n}$ is the Milnor hypersurface in $\mathbb{C}P^m \times \mathbb{C}P^n$

$$P(s) = \sum [\mathbb{C}P^n]\, s^n$$

$$F(s,t) = \sum a_{m,n}\, s^m t^n$$

There is an invariant of smooth $\mathbb{C}X$ projective algebraic varieties called **Todd genus** td. It is a cobordism invariant

disjoint unions $\longrightarrow$ sums

Cartesian products $\longrightarrow$ products

Can be defined in terms of Chern numbers.

It is a hom $\Omega^U = \pi_* MU \longrightarrow \mathbb{Z}$

see Hirzebruch: Topological methods in algebraic geometry.

$Td(\mathbb{C}P^n) = 1$. Redefine it as a gradual hom $\pi_* MU \to \mathbb{Z}[u] = R$

$$\downarrow$$

$$\mathbb{Z}[u, u^{-1}]$$

where $|u| = 2$

$$[\mathbb{C}P^n] \longmapsto u^n$$

What is the resulting FGL over R?

Recall the FGL logarithm

$$\log x = \sum_{n \geq 0} \frac{[\mathbb{C}P^n] x^{n+1}}{n+1} \in \pi_* MU \otimes \mathbb{Q}[[x]]$$

$$\log F(x, y) = \log(x) + \log(y)$$

MYSHENKO 1967.

It is an isomorphism with the additive FGL defined over $\pi_* MU \otimes \mathbb{Q}$.

$$\frac{d \log(x)}{dx} = P(x) = \sum [\mathbb{C}P^n] x^n$$

$$= \frac{1}{F_2(x,0)}$$ where

$$F_2(x,y) = \frac{\partial F(x,y)}{\partial y}$$

Applying the Todd genus to $\log x$ gives

$$\log x = \sum u^n \frac{x^{n+1}}{n+1}$$

Multiplicative FGL

$$\rightsquigarrow F(x,y) = x + y - u\,xy$$

If we set $u \mapsto 1$, we get

$$\log x = \sum_{n\geq 0} \frac{x^{n+1}}{n+1} = \sum_{n>0} \frac{x^n}{n}$$

$$= -\ln(1-x)$$

$MU_* \xrightarrow{td} R = \mathbb{Z}[u^{\pm 1}]$ defines the FGL / R given above.

Consider the function

$$X \longmapsto MU_*(X) \otimes_{MU_*} R =: K_*(X)$$

where R is an MU_*-module via td.
This functor has an exactness property needed to make it a generalized homology theory.
(this requires u to be invertible)
There is a spectrum K that represents this functor, i.e. $K_* X = \pi_*(X \wedge K)$.
It is the same as the K defined in terms of complex vector bundles.

This was known by 1965.

$$K_*(X) = MU_*(X) \otimes_{MU_*} K$$

CONNER-FLOYD

This all works equivariantly.

Replace K by $K_{\mathbb{R}}$

MU by $MU_{\mathbb{R}}$

C_2-equivariant spectra

Recall we know the differentials in slice SS for $MU_{\mathbb{R}}$.

There there an equivariant map

$MU_{\mathbb{R}} \longrightarrow K_{\mathbb{R}}$ underlain by the map $MU \to K$ related to the Todd genus. Hence we get a map of slice spectral sequences. The d_3 in the slice SS for $MU_{\mathbb{R}}$ maps to the d_3 in the slice SS for $K_{\mathbb{R}}$. Recall that

$$\Phi^{C_2} MU_{\mathbb{R}} \simeq MO \quad \text{and} \quad \pi_* MO$$

was determined by Thom 1954.