Want to prove the Hiep Theorem, which identifies in $M_{UR}$ and its relatives.

For $C_1 \subset C_2$, let $A_1 = S^0[C^p, S^{2p}, S^{3p}, \ldots]$. Then $\prod_x M_{UR}$ is refined by a map

$h_1$ from $A_1$.

Let $C_1 = C_{2^n} \supset C_2 = \text{Hand}A_n = N_{H}^{N_{G}}A$. Then $\prod_x N_{G}^{N_{G}}M_{UR}$ is refined by a map $h_1$ from $A_n$. 


Fix \( n \) and let \( A = A_n \) and
\[ N^G_H M_U R = M_U((G)) \]
\[ T_n^G M_U((G)) \] be a graded polynomial algebra with \( \text{deg} R = 2^n-1 \) in every positive even dimension.
Let \( M_{2d} \subset A \) be the wedge of all summands of dimension \( \geq 2d \).
\[ \rightarrow M_{2d+2} \rightarrow M_{2d+1} \rightarrow M_{2d-2} \rightarrow \cdots \rightarrow M_0 = A \]
\[ W_{2d} = M_{2d} / M_{2d+2} \]
wedge of all suspensions of dimension 2d

= wedge of \((2d)\)-dimensional slice cells.

Slice Theorem

\[
P^k_{\mathbb{A}} \mathbb{M}U((G)) = \begin{cases} \mathbb{W}^{2d} & \text{if } k = 2d \\ \times & \text{otherwise} \end{cases}
\]

Recall \(A\) is a "ring" and \(M_{2d}\) is a module over it. Let \(K_{2d} = \mathbb{M}U((G)) \mathbb{A} M_{2d}\)
e.g. $K_0 = MU((G)) \wedge M_{2d} = MU((G)) \wedge A$

$= MU((G))$

Ignoring the $G$-action, $K_{2d}$ is the $(2d-1)$-connected cover of $MU((G))$

Lemma: There are cofiber sequences

$K_{2d+2} \to K_{2d} \to K_{2d}/K_{2d+2}$

and

$K_{2d}/K_{2d+2} \to MU((G))/K_{2d+2} \to MU((G))/K_{2d}$
Let $R(\infty) = MU((G)) \rightarrow S^0$ (underline by $H2$). Then $K_{2d+2} / K_{2d+2} = R(\infty) \cap W_{2d}$.

Proof is formal.

We have a filtration of $MU((G))$

$$K_{2d+2} \rightarrow K_{2d} \rightarrow \cdots \rightarrow K_2 \rightarrow K_0 = MU((G))$$

and can identify each subquotient. Non-equivariantly there are the connective covers of $MU((G))$.

We want to show this is the
Slice filtration of $\text{MU}(G)$.

6.5 Reduction theorem $K_0 / K_2 \cong R_0$

is equivariantly equivir to $\mathbb{H}^2$.

Will show that Reduction Theorem implies Slice Theorem.

Lemma 6.6 $K_{2d+2} \to 2d$ (in the slice filtration, i.e. it is built out of slice cells of dim $\geq 2d$).

Proof: $K_{2d+2} = \text{MU}((G)) \wedge M_{2d+2}$
Clearly $M_{2d+2} \geq 2d$. Consider the class of $A$-modules $M$ for which $M_A \otimes M_{2d+2} \geq 2d$. It is closed under finite direct limits and extensions. It contains $\Sigma^k G / H_k \otimes A$ for $k \geq 0$ and $H$ is any subgroup. It contains every $(d)$-connected $A$-module. Hence it contains $MU((G))$, QED.
Lemma 6.17  If 6.5 holds then

\[ \frac{\text{MU}((6i))}{K_{2d+2}} \leq 2d, \text{ i.e.} \]

\[ \forall X > 2d, \exists [X, \text{MU}((6i))/K_{2d+2}] = 0. \]

Proof: by induction on d.

For d = 0, \text{MU}((6i))/K_2 = R(\infty).

6.5 implies this is \( \leq d \). A 0-slice exists. There is a cofiber sequence:

\[ \text{R}(6i) \cap W_{2d} \rightarrow \text{MU}((6i))/K_{2d+2} \rightarrow \text{MU}((6i))/K_{2d}. \]

By 2d-slice \( \leq 2d \) \( \leq 2d-2 \).
Proof that Reduction Theorem implies Slice Theorem: Consider

$$K_{2d+2} \rightarrow MU((G)) \rightarrow MU((G))/K_{2d+2}.$$ 

The 2-Lemma implies that

$$P^d MU((G)) = P^d MU((G)) = MU((G))/K_{2d+2}.$$ 

This means the odd slices vanish and the even slices are as
advertised. QED.