Note Title 11/20/2012

Want to prove the Slice Theorem, which identifies $\pi_*^u MV_{\mathbb{R}}$ and its relatives.

For $G = C_2$, let $A_1 = S^0[S^\rho, S^{2\rho}, S^{3\rho}, \dots]$

Then $\pi_*^u MV_{\mathbb{R}}$ is refined by a map f_1 from A_1

Let $G = C_{2^n} \supset C_2 = H$ and $A_n = N_H^G A_1$

Then $\pi_*^u N_H^G MV_{\mathbb{R}}$ is refined by a map f_n from A_n.

Warning: $f_n \neq N_H^G f_2$. TECHNICAL.

Fix n and let $A = A_n$ and

$N_H^G MU_{\mathbb{R}} = MU^{((G))}$

$\pi_*^u MU^{((G))}$ is a graded polynomial algebra with $|G|/2 = 2^{n-1}$ in every positive even dimension.

Let $M_{2d} \subset A$ be the wedge of all summands of dimension $\geq 2d$.

$$\cdots \to M_{2d+2} \longrightarrow M_{2d} \to M_{2d-2} \cdots \to M_0 = A.$$

$$W_{2d} = M_{2d} / M_{2d+2}$$

= wedge of all summands of dimension 2d
= wedge of (2d)-dimensional slice cells.

Slice Theorem

$$P^k_k MU^{((G))} = \begin{cases} W_{2d} \wedge H\underline{\mathbb{Z}} & \text{if } k = 2d \\ * & \text{otherwise} \end{cases}$$

Recall A is a "ring" and M_{2d} is a module over it.

Let $K_{2d} = MU^{((G))} \underset{A}{\wedge} M_{2d}$

e.g. $K_0 = MU^{((G))} \wedge_A M_0 = MU^{((G))} \wedge_A A$

$= MU^{((G))}$

Ignoring the G-action, K_{2d} is the $(2d-1)$-connected cover of $MU^{((G))}$

<u>Lemma</u> There are cofiber sequences

$$K_{2d+2} \to K_{2d} \to K_{2d}/K_{2d+2}$$

and

$$K_{2d}/K_{2d+2} \to MU^{((G))}/K_{2d+2} \to MU^{((G))}/K_{2d}$$

Let $R(\infty) = MU^{((G))} \wedge_A S^0$ (underlying HZ). Then $K_{2d}/K_{2d+2} = R(\infty) \wedge W_{2d}$

Proof is formal.

We have a filtration of $MU^{((G))}$

$$K_{2d+2} \hookrightarrow K_{2d} \hookrightarrow \cdots \hookrightarrow K_2 \hookrightarrow K_0 = MU^{((G))}$$

and can identify each subquotient.

Nonequivariantly these are the connective covers of $MU^{((G))}$.

We want to show this is the

Slice filtration of $MU^{((G))}$.

6.5 Reduction Theorem $K_0/K_2 = R_\infty$ is equivariantly equiv to $H\underline{\mathbb{Z}}$.

Will show that Reduction Thm implies Slice Thm.

HHR 2010 Lemma 6.6 $K_{2d+2} > 2d$ (in the slice filtration, i.e. it is built out of slice cells of dim $> 2d$.

Proof: $K_{2d+2} = MU^{((G))} \underset{A}{\wedge} M_{2d+2}$

Clearly $M_{2d+2} > 2d$. Consider the class of A-modules M for which $M \wedge_A M_{2d+2} > 2d$.
It is closed under filty direct limits and extensions. It contains $\Sigma^k G/H_+ \wedge A$ for $k \geq 0$ and H is any subgp. It contains every (-1)-connected A-module. Hence it contains $MU^{((G))}$. QED

Lemma 6.7 If 6.5 holds then $MU^{((G))}/K_{2d+2} \leq 2d$, i.e.

if $X > 2d$, $[X, MU^{((G))}/K_{2d+2}] = 0$.

Proof is by induction on d.

For $d = 0$, $MU^{((G))}/K_2 = R(\infty)$

6.5 says this is $H\underline{\mathbb{Z}}$, a 0-slice

There is a cofiber sequence

$$R(\infty) \wedge W_{2d} \longrightarrow MU^{((G))}/K_{2d+2} \longrightarrow MU^{((G))}/K_{2d}$$

$\underset{\text{2d-slice}}{\parallel}$ $\quad \leq 2d \quad \leq 2d-2$

$\leq 2d$ by induction

QED

Proof that Reduction Theorem implies Slice Theorem: Consider

$$K_{2d+2} \longrightarrow MU^{((G))} \longrightarrow MU^{((G))}/K_{2d+2}.$$

The 2 lemmas imply that

$$P^{2d+1} MU^{((G))} = P^{2d} MU^{((G))} = MU^{((G))}/K_{2d+2}.$$

This means the odd slices vanish and the even slices are as advertised. QED.