

$$G = C_{2^n} \quad MU^{(G)} = N_2^{2^n} MU_{\mathbb{R}} \\ = MU_{\mathbb{R}}^{(2^{n-1})}$$

We have generators

$$\begin{array}{c} \overline{m} \\ \downarrow \\ \mathbb{Z} \end{array} \in \pi_{\neq p} \left(i_{C_2}^G MU^{(G)} \right)$$

where i_H^G (G -spectrum X) for $H \subset G$

$= X$ regarded as an H -spectrum

There is a map

$$A = N_2^{2^n} S^0 [S^{p_2}, S^{2p_2}, S^{3p_2}, \dots] \rightarrow MU^{(G)}$$

where $S^1 P_2 \xrightarrow{\pi_A} i_2^{2^n} MU((G))$
 leads to

$$B = S^0 [S^1 P_2, S^2 P_2, \dots] \longrightarrow i_2^{2^n} MU((G))$$

Then apply $N_2^{2^n}$ and get

$$N_2^{2^n} B \longrightarrow N_2^{2^n} i_2^{2^n} MU((G)) = (MU((G))^{(2^n-1)})$$

)
 A

↓ multiplication
 $MU((G))$

The map $A \longrightarrow MU((G))$ refines
 $\pi_*^u MU((G))$. Consider the spectrum

$$R(\infty) = MU^{((G))} \wedge_A S^0$$

A is a ring over which $MU^{((G))}$ and S^0 are modules

Reduction Theorem (RT) $R(\infty) \simeq H\mathbb{Z}$
as G -spectra.

This implies the slice Theorem, which identifies the slices of $MU^{((G))}$

Recall the slice cells for a group G

$$\text{are } \left\{ \Sigma_{G/H}^{-1} S^{mPH}, G_+ \wedge_H S^{mPH} \vdash HCG, m \in \mathbb{Z} \right\}$$

Def A slice cell as above is
regular if it is not a decomposition
isotropic if $H \neq \emptyset$.

A slice is
cellular if it has the form $W \cdot H \cdot Z$
where W is a wedge of slice cells

A cellular
slice is

{ isotropic if W has no free summands
pure if W has no decompositions.

A G -spectrum X is isotropic/pure
if each of its slices is.

Example The slice theorem says
 $M/G^{(G)}$ is isotropic + pure.

Lemma 7.1 If X is pure (and isotropic)
and \tilde{W} is a wedge of regular (and isotropic)
slice cells then $X \times W$ is pure (and
isotropic).

Pf follows from definitions. Smearing
with a regular W preserves the
slice filtration (QED)

Lemma 7.2 Let $G = C_{2^n}$ and $H \subset G$ the

index \geq subgp. If RT holds for H ,
 then $i_H^{G_1} MV^{((G_1))}$ is pure and isotropic.

Proof Can show .

$$i_H^{G_1} MV^{((G_1))} = MV^{((H))} [H \cdot \bar{b}_1, H \cdot \bar{b}_2, \dots]$$

for certain generators $\bar{b}_i \in \Pi_{i, \mathbb{P}_2}^{i-2} (i_2^{-1} MV^{((G_1))})$

$$i_H^{G_1} MV^{((G_1))} = MV^{((H))} \wedge (\text{wedge of regular, isotropic slice cells})$$

Hence 7.2 follows from 7.1. QED

Prop 7.3 The reduction + slice thms for H imply that for the G_1 -spectrum $R(\mathfrak{a})$ defined above

$i_{H}^{G} R(\infty) = i_{H}^{G} H\mathbb{Z}$ for the
 G -spectrum $H\mathbb{Z}$.

Pf is formal:

Recall the isotropy separation
sequence of G -spectra

$$\begin{array}{c}
 E_{G_{2+}} = EP_{+} \longrightarrow S^{0} \longrightarrow EP \\
 \parallel \\
 E(G/H)_{+}
 \end{array}
 \quad (1)$$

We have a map $R(\infty) \longrightarrow H\mathbb{Z}$ which
we want to show to be an equivalence
smashing with (1) gives

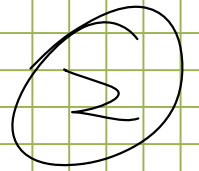
$$EP_n \cap R(\infty) \longrightarrow R(\infty) \longrightarrow EP_n \cap R(\infty)$$

$$b \downarrow \cong$$

$$g \downarrow$$

$$h \downarrow$$

$$EP_n \cap H \cong \longrightarrow H \cong \longrightarrow EP_n \cap H \cong$$



f is a G -equivalence because each cell in EP is induced up from H and we have $\exists \eta \exists$. Hence it suffices to show that h is a G -equivalence.

Back ① let $K \subset G$

$$(EP)^K = \begin{cases} EP \cong * & \text{for } K \neq G \\ \emptyset & \text{for } K = G \end{cases}$$

$$(\mathbb{E}P_+)^K = \begin{cases} S^0 & \text{for } K \neq G \\ \text{pt} & \text{for } K = G \end{cases}$$

$$(\tilde{\mathbb{E}P})^K = \begin{cases} * & \text{for } K \neq G \\ S^0 & \text{for } K = G \end{cases}$$

It follows that for any G -spectrum X

$$(\tilde{\mathbb{E}P} \wedge X)^K = \begin{cases} * & \text{for } K \neq G \\ ? & K = G \end{cases}$$

$$(\tilde{\mathbb{E}P} \wedge X)^G = \bigoplus_{\text{point}}^G X = \text{geometric fixed set by definition.}$$

Hence to show h in (2) is a
G-equivalence, it suffices to
show that $\underline{\Phi}^G g$ is an equivalence.

$$\text{i.e. } \underline{\Phi}^G R(\infty) \xrightarrow{\sim} \underline{\Phi}^G \# \underline{\Sigma}$$