Note Title 11/26/2012

$G = C_{2^n}$ $\quad MU^{((G))} = N_2^{2^n} MU_{\mathbb{R}}$

$= MU_{\mathbb{R}}^{(2^{n-1})}$

We have generators

$\overline{r}_j \in \pi_{j\rho}\left(i_{C_2}^{G} MU^{((G))}\right)$

where i_H^G (G-spectrum X) for $H \subset G$

$=$ X regarded as an H-spectrum

There is a map

$A = N_2^{2^n} S^0[S^{\rho_2}, S^{2\rho_2}, S^{3\rho_2}, \ldots] \longrightarrow MU^{((G))}$

where $S^{i\rho_2} \xrightarrow{\bar{r}_i} i_2^{2^n} MU^{((G))}$

leads to

$$B = S^0[S^{\rho_2}, S^{2\rho_2}, \dots] \longrightarrow i_2^{2^n} MU^{((G))}$$

Then apply $N_2^{2^n}$ and get

$$\begin{tikzcd}
N_2^{2^n} B \arrow[r] \arrow[d, equal] & N_2^{2^n} i_2^{2^n} MU^{((G))} = \left(MU^{((G))}\right)^{(\wedge 2^{n-1})} \arrow[d, "\text{multiplication}"] \\
A & MU^{((G))}
\end{tikzcd}$$

The map $A \longrightarrow MU^{((G))}$ refines $\pi_*^u MU^{((G))}$. Consider the spectrum

$$R(\infty) = MU^{((G))} \wedge_A S^0$$

A is a ring over which $MU^{((G))}$ and S^0 are modules

Reduction Theorem (RT) $R(\infty) \simeq H\underline{\mathbb{Z}}$ as G-spectra.

This implies the slice Theorem, which identifies the slices of $MU^{((G))}$.

Recall the slice cells for a group G are $\{\Sigma^{-1} G_+ \wedge_H S^{m\rho_H},\ G_+ \wedge_H S^{m\rho_H} : H \subset G,\ m \in \mathbb{Z}\}$

Def A slice cell as above is

regular if it is not a desuspension

isotropic if $H \neq e$.

A slice is

cellular if it has the form $W \wedge H\underline{\mathbb{Z}}$

where W is a wedge of slice cells

a cellular slice is

isotropic if W has no free summands

pure if W has no desuspensions.

A G-spectrum X is isotropic/pure if each of its slices is.

Example The slice theorem says $MU^{((G))}$ is isotropic + pure.

Lemma 7.1 If X is pure (and isotropic) and $\hat{W}$ is a wedge of regular (and isotropic) slice cells then $X \wedge W$ is pure (and isotropic).

Pf follows from definitions. Smashing with a regular W preserves the slice filtration. QED

Lemma 7.2 Let $G = C_{2^n}$ and $H \subset G$ the index 2 subgp. If RT holds for H, then $i_H^G MU^{((G))}$ is pure and isotropic.

Proof Can show

$$i_H^G MU^{((G))} = MU^{((H))}[H\cdot \bar{b}_1, H\cdot \bar{b}_2, \dots]$$

for certain generators $\bar{b}_j \in \pi_{j\rho_2}(i_2^{G} MU^{((G))})$

$$i_H^G MU^{((G))} = MU^{((H))} \wedge \left(\begin{array}{c}\text{wedge of regular}\\ \text{isotropic slice cells}\end{array}\right)$$

Hence 7.2 follows from 7.1. QED

Prop 7.3 The reduction + slice thms for H imply that for the G-spectrum $R(\infty)$ defined above

$i_H^G R(\infty) = i_H^G H\underline{\mathbb{Z}}$ for the G-spectrum $H\underline{\mathbb{Z}}$.

Pf is formal.

Recall the isotropy separation sequence of G-spectra

$$EC_{2+} = EP_+ \longrightarrow S^0 \longrightarrow \tilde{E}P \qquad (1)$$

$$\|$$

$$E(G/H)_+$$

We have a map $R(\infty) \longrightarrow H\underline{\mathbb{Z}}$ which we want to show to be an equivalence. Smashing with (1) gives

$$\begin{tikzcd}
EP_+ \wedge R(\infty) \arrow[r] \arrow[d, "f"', "x"] & R(\infty) \arrow[r] \arrow[d, "g"'] & \tilde{E}P \wedge R(\infty) \arrow[d, "h"'] \\
EP_+ \wedge H\underline{\mathbb{Z}} \arrow[r] & H\underline{\mathbb{Z}} \arrow[r] & \tilde{E}P \wedge H\underline{\mathbb{Z}}
\end{tikzcd}$$

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f is a G-equivalence because each cell in EP is induced up from H and we have $7 \wedge 3$. Hence it suffices to show that h is a G-equivalence.

Back 1 let $K \subset G$

$$(EP)^K = \begin{cases} EP \simeq * & \text{for } K \neq G \\ \emptyset & \text{for } K = G \end{cases}$$

$$(EP_+)^K = \begin{cases} S^0 & \text{for } K \neq G \\ pt & \text{for } K = G \end{cases}$$

$$(\tilde{EP})^K = \begin{cases} * & \text{for } K \neq G \\ S^0 & \text{for } K = G \end{cases}$$

It follows that for any G-spectrum X

$$(\tilde{EP} \wedge X)^K = \begin{cases} * & \text{for } K \neq G \\ ? & K = G \end{cases}$$

$(\tilde{EP} \wedge X)^G = \Phi^G X$ = geometric fixed point set by definition.

Hence to show h in ② is a G-equivalence, it suffices to show that $\Phi^G g$ is an equivalence.

i.e. $\Phi^G R(\alpha) \xrightarrow{\sim} \Phi^G H\underline{\Sigma}$