Note Title 11/30/2012

We have reduced the Slice + Reduction Theorems to a smaller statement

Recall $M = MU^{((G))} = N_2^{\Sigma_n} MU_{\mathbb{R}}$, $G = C_{2^n}$

$$M_i = M / (G \cdot \bar{r}_i)$$

$$R(\infty) = M_1 \wedge_M M_2 \wedge_M M_3 \wedge_M \cdots$$

It suffices to show that the map $\Phi^G R(\infty) \to \Phi^G H\underline{\mathbb{Z}}$ is an equiv

We know $\pi_* \Phi^G H\underline{\mathbb{Z}} = \mathbb{Z}/2[b]$ for $b \in \pi_2$

Let $c_k = 2^{k-1}$ and $M_{(k)} = M_{c_k}$

It suffices to show that for each $k > 0$, b^{2^k} is in the image of the map $\Phi^G M_{(k)} \longrightarrow \Phi^G H\underline{\mathbb{Z}}$.

Recall the isotropy separation sequence

$$E\mathcal{P}_+ \wedge X \longrightarrow X \longrightarrow \tilde{E}\mathcal{P} \wedge X$$

where $E\mathcal{P} = E(G/G') = EC_2$, where G' is the subgp of index 2.

$\mathcal{P}$ denotes the family of proper subgps of G.

$$(\tilde{E}\mathcal{P} \wedge X)^H = \begin{cases} * & \text{if } H \neq G \\ \Phi^G X & H = G \end{cases}$$

Lemma Let T be a G-space with $T_0 = T^G$. Then for any G-spectrum X, the restriction map

$$[T, \tilde{E}\mathcal{P} \wedge X] \longrightarrow [T_0, \tilde{E}\mathcal{P} \wedge X]$$

is an isomorphism.

Proof Since $(\tilde{E}\mathcal{P} \wedge X)^H = *$ for $H \neq G$,

$$\underline{\pi}_*(\tilde{E}\mathcal{P} \wedge X)(G/H) := \pi_*(\tilde{E}\mathcal{P} \wedge X)^H = 0.$$

It follows that if W is a G-CW cx built of cells of the form $G_+ \wedge_H D^n$ for $H \neq G$, then

$$[W, \tilde{E}\mathcal{P} \wedge X] = 0$$

Note that we have a cofiber seq

$$T_0 \longrightarrow T \longrightarrow W \quad \text{for } W \text{ as above}$$

The result follows. QED

<u>Prop 7.10</u> Each $b^{2^{k-1}}$ is in the image of

$$\pi_* \Phi^G M_{(k)} \longrightarrow \pi_* \Phi^G H\underline{\mathbb{Z}} \quad \text{for each } k > 0.$$

<u>Pf</u> Apply the lemma to

$$T = S^{1+c_k\rho_G}$$

$$T_0 = T^G = S^{1+c_k} = S^{2^k}$$

We get $\pi_{1+c_k\rho_G}(\tilde{E}P, X) = \pi_{2^k}(\tilde{E}P, X)$

Consider the element

$$x = N_2^{2^n}\, \bar{m}_{C_R} \in \pi^{G}_{C_R \rho_G} MU^{((G))}$$

We will chase the diagram

$$\begin{tikzcd}
EP_+ \wedge M \arrow[r] \arrow[d] & M \arrow[r] \arrow[d] & \tilde{E}P \wedge M \arrow[d] \\
EP_+ \wedge M_{(R)} \arrow[r] \arrow[d] & M_{(R)} \arrow[r] \arrow[d] & \tilde{E}P \wedge M_{(R)} \arrow[d] \\
EP_+ \wedge H\underline{\mathbb{Z}} \arrow[r] & H\underline{\mathbb{Z}} \arrow[r] & \tilde{E}P \wedge H\underline{\mathbb{Z}}
\end{tikzcd}$$

x has trivial image in

$$\pi^{G}_{C_k \rho_G} \widetilde{EP} \wedge M \overset{\text{LEMMA}}{=} \pi^{G}_{C_k} \widetilde{EP} \wedge M = \pi_{C_k} \Phi^{G} M = \pi_{2^k - 1} MO$$

Prop 7.12 Any choice of y has nontrivial image in $\pi^{G}_{C_k \rho_G} EP_+ \wedge H\mathbb{Z}$.

$\bar{y}$ has trivial image in $\pi^{G}_{C_k \rho_G} M_{(k)}$,

so it comes from a class

$\bar{\bar{y}} \in \pi^{G}_{1 + C_k \rho_G} \widehat{EP} \wedge M_{(k)}$

$$\begin{tikzcd}
\pi^{G}_{1+c_k\rho_G}\tilde{E}P\wedge M_{(k)} \arrow[rr] \arrow[dd] \arrow[dr] & & \pi^{G}_{c_k\rho_G}EP_+\wedge M_{(k)} \arrow[dd] \\
& \pi^{G}_{2^k}\tilde{E}P\wedge M_{(k)} = \pi_{2^k}\Phi^{G}M_{(k)} \arrow[dd] & \\
\neq 0 \quad \pi_{1+c_k\rho_G}\tilde{E}P\wedge H\underline{\mathbb{Z}} \arrow[rr] \arrow[dr] & & \pi_{c_k\rho_G}EP_+\wedge H\underline{\mathbb{Z}} \quad \neq 0 \\
& \pi^{G}_{2^k}\tilde{E}P\wedge H\underline{\mathbb{Z}} &
\end{tikzcd}$$

$$\pi^{G}_{2^k}\tilde{E}P\wedge H\underline{\mathbb{Z}} = \pi_{2^k}\Phi^{G}H\underline{\mathbb{Z}} = \mathbb{Z}/2 \ni b^{2^{k-1}}$$

Hence $b^{2^{k-1}}$ is in the image of $\pi_{2^k}\Phi^{G}M_{(k)}$ as desired. The theorem hence follows from 7.12.

Proof on Monday.