We have reduced the Slice + Reduction Theorem to a smaller statement.

Recall \( M = M \cup \{0\} = N \sum_{i=1}^{m} M \cup \{i\} \quad \Rightarrow \quad G = C_2^n \)

\[ M_i = M / (C_i, \circ M_i) \]

\( R(x) = M_1 / M_2 / M_3 / \cdots \)

It suffices to show that the map \( \phi^\alpha R(x) \to \bar{G}, H \geq \) is an equiv.

We know \( \prod_{x} \bar{G}, H \geq = \mathbb{Z}/2[\bar{b}] \) for \( b \in \mathbb{T}_2 \)

Let \( C_R = 2^k - 1 \) and \( M(\alpha) = M \circ C_R \)
It suffices to show that for each $k > 0$, $\mathbb{P}^k$ is in the image of the map $\mathcal{O}_{G_0, M_{(k)}} \to \mathcal{O}_{G_1, H_{(2)}}$.

Recall the isotropy separation sequence

$$\mathcal{E}P \cap x \longrightarrow x \longrightarrow \mathcal{E}P \cap x$$

where $\mathcal{E}P = \mathcal{E}(G/G') \cong \mathcal{E}_x$, where $G'$ is the subgroup of index 2, $P$ denotes the family of proper subgroups of $G$.

$$(\mathcal{E}P \cap x)^H = \begin{cases} \bigcup \mathcal{O}_{G_1, x} & \text{if } H \neq G, \\ \mathcal{O}_{G_0, x} & \text{if } H = G \end{cases}$$
Lemma: Let $T$ be a $G$-space with $T_0 = T^{G_0}$. Then for any $G$-spectrum $X$, the restriction map

$$[T, E^{P_n} X] \to [T_0, E^{P_n} X]$$

is an isomorphism.

Proof: Since $(E^{P_n} X)^H = X$ for $H \neq G$, we have

$$\pi_* (E^{P_n} X)(G/H) = \bigoplus H \neq G \pi_* (E^{P_n} X)^H = 0.$$ 

It follows that if $W$ is a $G$-CW complex limit of cells of the form

$$C_{n+1} \cdot D^n \quad \text{for} \quad H \neq G,$$

then

$$[W, E^{P_n} X] = 0.$$
Note that we have a cofiber seq
\[ T \rightarrow T \rightarrow W \] for \( W \) as above.

The result follows. QED

Prop 2.10: Each \( b^{2k-1} \) is in the image of 
\[ T_x \otimes M(k) \rightarrow T_x \otimes \mathcal{O}_x \mathbb{H} \] for each \( k > 0 \).

Proof: Apply the lemma to 
\[ T = \overline{s}^{1+C_k P_{g_2}} \]
\[ T_0 = T G_{n'} = \overline{s}^{1+C_k} = \overline{s}^{2k} \]

We get 
\[ \overline{T}^{1+C_k P_{g_2}} (E_{P_{n'}X}) = \overline{T}^{2k} (E_{P_{n'}X}) \]
Consider the element
\[ X = \mathbb{N}^{\mathbb{N}} \mathbb{N} \in \prod_{x \in \mathbb{N}} \mathbb{M} \mathbb{V} \]
We will chase the diagram

\[ \begin{array}{ccc}
    Y & \xrightarrow{EP_+ \cap M} & M \\
    \downarrow & & \downarrow \\
    EP_+ \cap M_{(k)} & \xrightarrow{M_{(k)}} & EP_+ \cap M_{(k)} \\
    \downarrow & & \downarrow \\
    EP_+ \cap \mathbb{I} \mathbb{Z} & \xrightarrow{\mathbb{I} \mathbb{Z}} & EP_+ \cap \mathbb{I} \mathbb{Z} \\
\end{array} \]

\[ \begin{array}{ccc}
    X & \xrightarrow{EP_+} & 0 \\
    \downarrow & & \downarrow \\
    EP_+ \cap M & \xrightarrow{EP_+ M} & EP_+ \cap M \\
    \downarrow & & \downarrow \\
    EP_+ \cap \mathbb{I} \mathbb{Z} & \xrightarrow{EP_+ \mathbb{I} \mathbb{Z}} & EP_+ \mathbb{I} \mathbb{Z} \\
\end{array} \]

\[ x \text{ has trivial image in} \]
Lemma
\[ \prod_{i \in I} \mathbb{E}P_i M = \prod_{i \in I} \mathbb{E}P_i M = \prod_{i \in I} \mathbb{E} \mathbb{P}_i M = \prod_{i \in I} \mathbb{P}_i M \]

Prop 7.12: Any choice of \( y \) has nontrivial image in \( \prod_{i \in I} \mathbb{E}P_i M \).

\( y \) has trivial image in \( \prod_{i \in I} \mathbb{E}P_i M_{(R)} \), so it comes from \( y \) class,

\[ y \in \prod_{i \in I} \mathbb{E}P_i M_{(R)} \]
Hence $t^2$ is in the image of $\pi_{2n} \bar{G} M_k(k)$ as desired. The Theorem
hence fellows from 7.12.
Proof on Monday.