Recall we want to prove the slice differentials theorem.

In the slice $SS$ for $\pi_{12}$ and $KR$, we have $d_2 (ux) = a^3 x^3$.

We will derive this from a more general theorem about slice $S$ for $MV_{1R}$.

Recall $MV_{1R}$ is constructed in the Thom spaces $MV(n)$, each in a $G_r$-bundle (where $G_r = C_r$).
action by complex conjugation, so \( MU(n)^\mathbb{C} = MO(n) \).
It follows that \( \pi_* MVR = MO \).
This fact is crucial for us.
Recall \( \pi_* M O = \mathbb{Z}[y_1, y_2, y_3, \ldots ] \) with \( \dim y_i = 1 \)
\[ \pi_* M U = \mathbb{Z}[x_1, x_2, \ldots ] \] with \( \dim x_i = 2i \).
\( y_i \) (or \( x_i \)) is represented by a real (complex) manifold of
Amicicán (ex diménsion) i.
The complex $X_i$ representing $X_i$
    can be chosen so that
$X_i$ represents $y_i$ for $i \neq 2^n - 1$.
In the algebra $SS$ for MVR
we have
\[ a = a_6 \in E_2^{13,1-6} (G/67) \]  with $2a_0 = 0$
\[ M = M_{20} \in E_2^{0,2-26} (6/1G) \]
\[ X_i \in E_2^{0,4+i6} (G_i/G_i) \]  for $i>0$
$E_2^{*} (G/67)$ is the ring generated by
the elements $E^*_2(C/\mathbb{G})$ is spanned by
\[ \{ \lambda^k m^N : \|N\| = k + 2m \} \]
where $N = (n_1, n_2, \ldots)$ with $m \leq n_i < \infty$
\[ \lambda^N = \lambda^{n_1} \lambda^{n_2} \cdots \]
\[ \|N\| = \frac{1}{2} \dim \lambda^N = \sum_{i \geq 0} i \cdot n_i. \]

We know that $\Sigma C_{MV_{IR}}$ can be computed by the 2-graded portion of $\lambda^N$. $E^*_2(C/\mathbb{G})$. 
Let \( w = a^{-2} \cdot x \in a^{-1} E^{-2,0}_2 (G / G) \)

\( y_i = a^i \cdot x_i \in E^{i, \Sigma i}_2 (G / G) \)

\[ a^i E^{k, k}_2 (G / G) = \frac{1}{2} \left[ w \cdot y_1 \cup y_2 \cup y_3 \cup \cdots \right] \]

\( y_i \) is permanent cycle

\( w \) need not be

\[ d_2 (w) = y_1 \]

\[ d_2 (w^2) = 0 \]

\[ d_7 (w^3) = y_3 \]

\[ d_{15} (w^4) = y_7 \text{ etc.} \]
We know \( y_1, y_2, y_3, \ldots \) etc. must be killed in the \( S_5 \). This means:

\[
y_2^{n-1} = d_{2_{n+1}}(w^{2_{n-1}}) \quad \text{for each} \quad n > 0.
\]

i.e.

\[
d_{2_{n+1}}(a^{-2^n} w^{2_{n-1}}) = a^{2^{-1}} x_{2^{-1}}^{n-1} \quad \text{for each} \quad n > 0.
\]

\[
a^{-2^n} d_{2_{n+1}}(w^{2_{n-1}}) = a^{2^{-n-1}} x_{2^{-1}}^{n-1}
\]

\text{Slice differentials}\quad \text{Theorem}\quad \text{modul...} a\text{-torsion}
e.g. \( d_3 (u) = a^3 x_1 \)
\( d_7 (u^2) = a^7 x_3 \)
\( d_{15} (u^4) = a^{15} x_7 \) etc

This completely determines the behavior of the slice SS for \( M(\mathbb{R}) \).

e.g.
\( d_3 (2u) = 2a^3 x_1 = 0 \)
2u is a permanent cycle.
\( d_3 u^2 = 2u d_3 u = 2u a^3 x_1 = 0 \)
\( d_{15} u^2 \) can be shown to vanish
by inspection

\[ d^2 \equiv a^2 \mod a \text{ a-torsion} \]

but there is no a-torsion in that bidegree.