Note Title 11/5/2012

Recall we want to prove the slice differentials theorem

In the slice SS for $bu_{\mathbb{R}}$ and $K_{\mathbb{R}}$ we have $d_3(u x) = a^3 x^3$.

Will derive this from a more general theorem about slice SS for $MU_{\mathbb{R}}$.

Recall $MU_{\mathbb{R}}$ is constructed in terms of the Thom spaces $MU(n)$.

Each is a G-space (where $G = C_2$)

action by complex conjugation, so $MU(n)^G = MO(n)$.

It follows that $\Phi^G MU_{\mathbb{R}} = MO$.

This fact is pivotal for us.

Recall $\pi_* MO = \mathbb{Z}/2[y_2, y_4, y_5, y_6, y_8 \cdots]$ with $\dim y_i = i$

$= \mathbb{Z}/2[y_i : i \neq 2^n - 1]$

$\pi_* MU = \mathbb{Z}[x_1, x_2 \cdots]$ with $\dim x_i = 2i$.

y_i (x_i) is represented by a real (complex) manifold of dimension (cx dimension) i.

The complex X_i representing x_i can be chosen so that X_i^G represents y_i for $i \neq 2^n - 1$.

In the slice SS for $MV_{\mathbb{R}}$ we have

permanent cycles

$$a = a_\sigma \in E_2^{1,1-\sigma}(G/G) \quad \text{with } 2a=0$$

$$u = u_{2\sigma} \in E_2^{0,2-2\sigma}(G/G)$$

$$x_i \in E_2^{0,i+i\sigma}(G/G) \quad \text{for } i>0$$

$E_2^{*,\star}(G/G)$ is the ring generated by these elements:

$E_2^{*,\star}(G/G)$ is spanned by

$$\{a^k u^m x^N : \|N\| = k + 2m\}$$

where $N = (n_1, n_2, \ldots\ldots)$ with $n_i \geq 0$, $\sum n_i < \infty$

$$x^N = x_1^{n_1} x_2^{n_2} \ldots\ldots$$

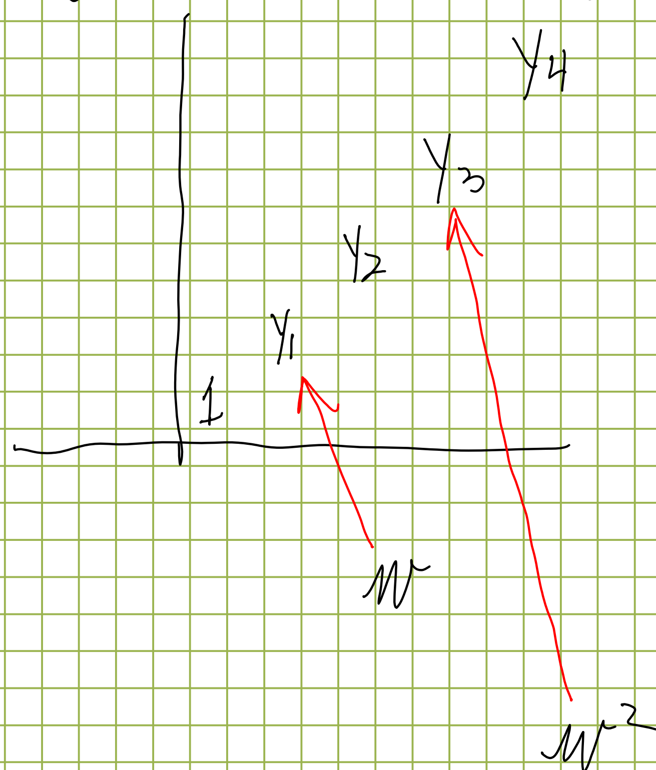
$$\|N\| = \frac{1}{2} \dim x^N = \sum_{i>0} i\, n_i .$$

We know that $\pi_\star \Phi^G MU_{\mathbb{R}}$ can be computed by the $\mathbb{Z}$-graded portion of $a^{-1} E_2^{*,\star}(G/G)$.

Let $w = a^{-2}u \in a^{-1}\underline{E}_2^{-2,0}(G/G)$

$$y_i = a^i x_i \in \underline{E}_2^{i,2i}(G/G)$$

$$a^{-1}\underline{E}_2^{*,*}(G/G) = \mathbf{Z}/2[w;\, y_1, y_2, y_3, \dots]$$



y_i is permanent cycle

w need not be

$d_3(w) = y_1$

$d_3(w^2) = 0$

$d_7(w^2) = y_3$

$d_{15}(w^4) = y_7$ etc

We know y_1, y_3, y_7 etc must be killed in the SS. This means

SLICE DIFF

$$y_{2^n-1} = d_{2^{n+1}-1}\left(u^{2^{n-1}}\right) \quad \text{for each } n>0.$$

i.e.

$$d_{2^{n+1}-1}\left(a^{-2^n} u^{2^{n-1}}\right) = a^{2^n-1} \, x_{2^n-1} \quad \text{for each } n>0$$

$$a^{-2^n} \, d_{2^{n+1}-1}\left(u^{2^{n-1}}\right) = a^{2^n-1} \, x_{2^n-1}$$

SLICE DIFFERENTIALS THEOREM

modulo a-torsion

$$d_{2^{n+1}-1}\left(u^{2^{n-1}}\right) = a^{2^{n+1}-1} \, x_{2^n-1} \quad \text{e.g. mod } 2$$

e.g. $d_3(u) = a^3 \chi_1$

$d_7(u^2) = a^7 \chi_3$

$d_{15}(u^4) = a^{15} \chi_7$ etc

This completely determines the behavior of the slice SS for $MU_{\mathbb{R}}$. e.g.

$d_3(2u) = 2a^3\chi_1 = 0$

$2u$ is a permanent cycle.

$d_3 u^2 = 2u\, d_3 u = 2u\, a^3 \chi_1 = 0$

$d_5 u^2$ can be shown to vanish

by inspection

$d_7 x^2 = a^7 \chi_3$ mod a-torsion

but there is no a-torsion in that bidegree.