Note Title 12/12/2012

Review of strategy. There is spectrum Ω, derived from $N_2^8 MU_{\mathbb{R}}$, with 3 properties

1) Detection Theorem: If $\theta \in \pi_{2^{j+1}-2} S^0$ exists, it has a nontrivial image in $\pi_* \Omega$

2) Periodicity Theorem $\Sigma^{256} \Omega \simeq \Omega$

so $\pi_i \Omega$ depends only on i mod 256

3) <u>Gap Theorem</u> $\pi_{-2}\Omega = 0$.

2) and 3) imply $\pi_{2^{j+1}-2}\Omega = 0$

for $j \geq 7$. 1) implies $\not\exists\, \theta_j$ for $j \geq 7$.

For 2) recall $K_{\mathbb{R}}$ with $G = C_2$

$2a = 0$ $\quad a = a_6 \in \underline{E}_2^{1,1-6}(G/G)$

$x = x_1 \in \underline{E}_2^{0,1+6}(G/G)$ $\Big\}$ perm cycles

$u = u_{26} \in \underline{E}_2^{0,2-26}(G/G)$ not perm.

$d_3(ux^2) = a^3x^3$ Slice differential

Hence inverting x gives

$d_3(x^{-1}u) = a^3$ so $E_4^{s,*} = 0$ for $s \geq 3$

(vanishing line)

$x^4 u^2$ is a permanent cycle

$\in E_2^{0,8}$

This leads to periodicity of dim 8.

These statements also make sense in $MV_{\mathbb{R}}$. Inverting x gives an 8-periodic C_2-spectrum.

We want a similar statement about $MV^{((G))} = N_2^{2^n} MV_{\mathbb{R}}$ where

$G = C_{2^n}$. We need to invent an element that will make a power of $u_{2\rho}$ a perm cycle, where $\rho = \rho_G$.

In the slice SS for $MU_{\mathbb{R}}$ we have a, u as before and $G = C_2$

$x_i \in \pi_{i\rho} MU_{\mathbb{R}}$ $\rho = \rho_{C_2}$

$a^i x_i = y_i \in \underline{E}_2^{i, 2i}(G/G)$ perm cycle

Let $w = a^{-2} u$ (formally)

$$d_{2^{n+1}-1}\left(w^{2^{k-1}}\right) = y_{2^k-1}$$

$$d_{2^{n+1}-1}\left(u^{2^{k-1}}\right) = a^{2^{k+1}-1}\, x_{2^k-1}$$

Inverting some x_{2^k-1} makes u^{2^k} a perm cycle.

What happens in $N_2^{2^n} MU_{\mathbb{R}} = MU^{((G))}$ for $G = C_{2^n}$

$N_2^{2^n} x_1 \in \pi_{1\rho_G}(G/G)$ and $E_2^{0,1\rho_G}(G/G)$

Let $\bar{\rho}_G = \rho_G - 1$ = reduced regular

$$f_i = a^{i}_{\rho_G} u_2^{2^n} x_i \in E_2^{i(g-1),\, i}(G/G)$$

ref.

where $g = |G| = 2^n$

$a = a_\sigma$ where σ is the sign rep. of G.

$u = u_{2\sigma}$

Thm 9.9 $\quad d_r\, u^{2^{k-1}} = a^{2^k} f_{2^k-1}$

where $r = 1 + (2^k - 1) g$.

SLICE DIFFERENTIALS THEOREM

$$\bar{\mathfrak{d}}_k^{G} = N_2^{2^n} x_{2^k-1}\, \epsilon\, \pi_{(2^k-1)\rho_G} MU^{((G))}$$

$$f_{2^k-1} = a_{\bar\rho}^{2^k-1}\, \bar{\mathfrak{d}}_k^{G}$$

Inverting $\bar{\mathfrak{d}}_k^{G}$ makes $u_{2\sigma}^{2^k}$ a permanent cycle.

We need to make a power of $u_{2\bar\rho}$ a permanent cycle.

divisible by $u_{2\sigma}$

More calculation is required.

Note $\rho_{C_8} = 1 + \sigma + (3$ rotations of $\mathbb{R}^2)$

of the 3 rotations, one has order 4 and 2 have order 8

For $H \subset G$ let

$$\chi_1^H \in \pi_{1\rho_2}^{C_2} MU^{((H))} \hookrightarrow \pi_{1\rho_2}^{C_2} MU^{((G))}$$

$h = |H|$

$$\bar{\partial}_k^H = N_2^h\left(\chi_{2^k-1}^H\right) \in \pi_{(2^k-1)\rho_H}^H MU^{((G))}$$

$$\Delta_k^H = u_{2(2^k-1)\rho_H}\left(\bar{\partial}_k^H\right)^2 \in E_2^{0,*}$$

Thm 9.12 Suppose D is divisible by $\bar{\partial}^H_{g/h}$ for all $e \neq H \subset G$.

Inverting it makes $u^{2^{g/2}}_{2\rho_G}$ a permanent cycle.

Cor 9.13 Inverting D as above makes $\Delta_1^G = u^{2^{g/2}}_{2\bar{\rho}_G}(\bar{a}_1^G)^{2 \cdot 2^{g/2}}$ a permanent cycle.

Thm 9.15 (Periodicity)

Let $M = D^{-1} MU^{((G))}$ (or any module over it)

Then multiplication by $(\Delta_1^G)^{2g/2}$ gives an equiv of underlying spectra

$$\Sigma^{2g \cdot 2^{g/2}} i_0^* M \xrightarrow{\simeq} i_0^* M$$

and hence

$$\Sigma^{\text{same}} M^{hG} \xrightarrow{\simeq} M^{hG}$$

NOTE

$g = \vert G \vert$	2	4	8	16	...
$2g \cdot 2^{g/2}$	8	32	256	2^{13}	

For $G = C_8$, D can be

$$D = N_2^8\left(\bar{\partial}_4^{C_2}\right) N_4^8\left(\bar{\partial}_2^{C_4}\right)\left(\bar{\partial}_1^{C_8}\right)$$

$$\in \prod_{19\rho_8} MU^{((G))}$$

$$19 = (2^1 - 1) + (2^2 - 1) + (2^4 - 1)$$