Note Title 12/3/2012

Recall

Prop 7.10 Let $b \in a_{2\sigma}^{-1} u_{2G} \in \pi_2 \Phi^G H\underline{\mathbb{Z}}$

For each $k > 0$, b^{2^k-1} is in the image of

$$\begin{tikzcd}
\pi_\star \Phi^G M_{(k)} \arrow[rr] \arrow[dr] & & \pi_\star \Phi H\underline{\mathbb{Z}} \\
& \pi_\star \Phi^G R(\infty) \arrow[ur] &
\end{tikzcd}$$

Proving this involves chasing a diagram, the smash of $EP_+ \to S^0 \to \tilde{E}P$ and

$$M \to M_{(k)} \to H\underline{\mathbb{Z}}$$

some y

$$\begin{tikzcd}
 & & x \arrow[rr] \arrow[dd] & & 0 \\
EP_+ \wedge M \arrow[r] \arrow[d] & M \arrow[r] \arrow[d] & & \widetilde{EP} \wedge M \arrow[d] & \\
EP_+ \wedge M_{(k)} \arrow[r] \arrow[d] & M_{(k)} \arrow[r] \arrow[d] & 0 & \widetilde{EP} \wedge M_{(k)} \arrow[d] & \\
EP_+ \wedge H\underline{\mathbb{Z}} \arrow[r] & H\underline{\mathbb{Z}} \arrow[r] & & \widetilde{EP} \wedge H\underline{\mathbb{Z}} &
\end{tikzcd}$$

$y \to\ \stackrel{?}{=} 0$

$(T =\ )$

rows are cofiber sequences

$$x = N_2^{2^n} \overline{m}_{C_k} \in \pi_V^{G} M$$

Prop 7.12 Any choice of y has a $\neq 0$ image in $\pi_V\, EP_+ \wedge H\underline{\mathbb{Z}}$

This implies the Reduction Theorem.

$$\begin{tikzcd}
EP_+ \wedge M \arrow[rr] \arrow[dr] & & EP_+ \wedge H\underline{\mathbb{Z}} \\
 & EP_+ \wedge R(C_k) \arrow[ur] &
\end{tikzcd}$$

let $X = R(C_k)$

Will consider the slice tower for X
We know everything about its restriction to H (by inductive hypothesis) and hence its smash product with EP_+

Prop 7.14 (i) $\pi_V\ EP_+ \wedge P_m^m X = 0$ for $0 < m < 2d$

(ii) there is an exact sequence

$$\pi_V\ EP_+ \wedge P_d X \longrightarrow \pi_V EP_+ \wedge X \ni y_k = \operatorname{Im} y$$
$$\downarrow$$
$$\pi_V\ EP_+ \wedge H\mathbb{Z}.$$

slice connective cover

Pf (i) follows by induction on $|G|$

(ii) The map $\pi_V\ EP_+ \wedge P_d X \longrightarrow \pi_V\ EP_+ \wedge P_1 X$

is onto we have a cofiber sequence

$$EP_+ \wedge (P_1 X \longrightarrow X \longrightarrow P_0^0 X)$$

and we know $EP_+ \wedge P_0^0 X = EP_+ \wedge H\underline{\mathbb{Z}}$ QED

It suffices to show $1/k$ is not in the image of $\pi_V EP_+ \wedge P_d X$.

$$\begin{array}{c} \pi_V EP_+ \wedge P_d X \\ \parallel \\ \pi_V EP_+ \wedge P_1 X \end{array}$$

Let $\gamma \in G$ be a generator.

<u>Prop 7.15</u>

$$\begin{tikzcd}
 & \pi_V P_d X \arrow[d] & & \\
\pi_V^G EP_+ \wedge P_d X \arrow[ur] \arrow[r, "\alpha"] & \pi_V^G X \arrow[r, "i_0"] & \pi_d^u X
\end{tikzcd}$$

The image of

is contained in the image of $1-\gamma$, using the action of $\mathbb{Z}G$ on $\pi_*^{u} X$.

We will show that $\operatorname{im} \alpha$ is contained in the image of $\operatorname{Tr}_H^G : \pi_V^H X \to \pi_V^G X$

$$\begin{array}{ccc} \pi_V^H X & \longrightarrow & \pi_V^G X \\ \| & & \| \\ \underline{\pi}_V X(G/H) & & \underline{\pi}_V X(G/G) \end{array}$$

Will also show the $\gamma(i_0^* y_k) = -i_0^* y_k$

so $i_0^* y_k$ is not in the image of $(1-\gamma)$

y makes $i_0(N_2^{2^n} \bar{\mu}_{C_k}) =: z$ and $\gamma(z) = (-1)^{C_k} z$
$= -z$.

Prop 7.16 Let $Y \geq 0$ (i.e. (-1)-slice connected) be a G-spectrum. Then the image of

$$\underline{\pi}_0\, EP_+ \wedge Y\,(G/G) \longrightarrow \underline{\pi}_0\, Y\,(G/G)$$

is in image of Tr_H^G.

Pf Since Y = (-1)-slice-connected

$$\begin{tikzcd} \pi_0^G\, C_{2+} \wedge Y \arrow[r, "onto"] & \pi_0^G\, EP_+ \wedge Y \arrow[r] & \pi_0^G\, Y \\ \pi_0^H\, Y \arrow[u, phantom, "||"] \arrow[urr, bend right, "Tr_H^G"'] & & \end{tikzcd}$$

QED

Let $Y = \Sigma^{-V} P_d\, X$ and get

Cor 7.17 Image of $\pi_V^G\, EP_+ \wedge P_d\, X \longrightarrow \pi_V^G\, P_d\, X$

is in the image of Tr_H^G.

Cor 7.18 Same for $\pi_V^G \ \overline{\Sigma}$.