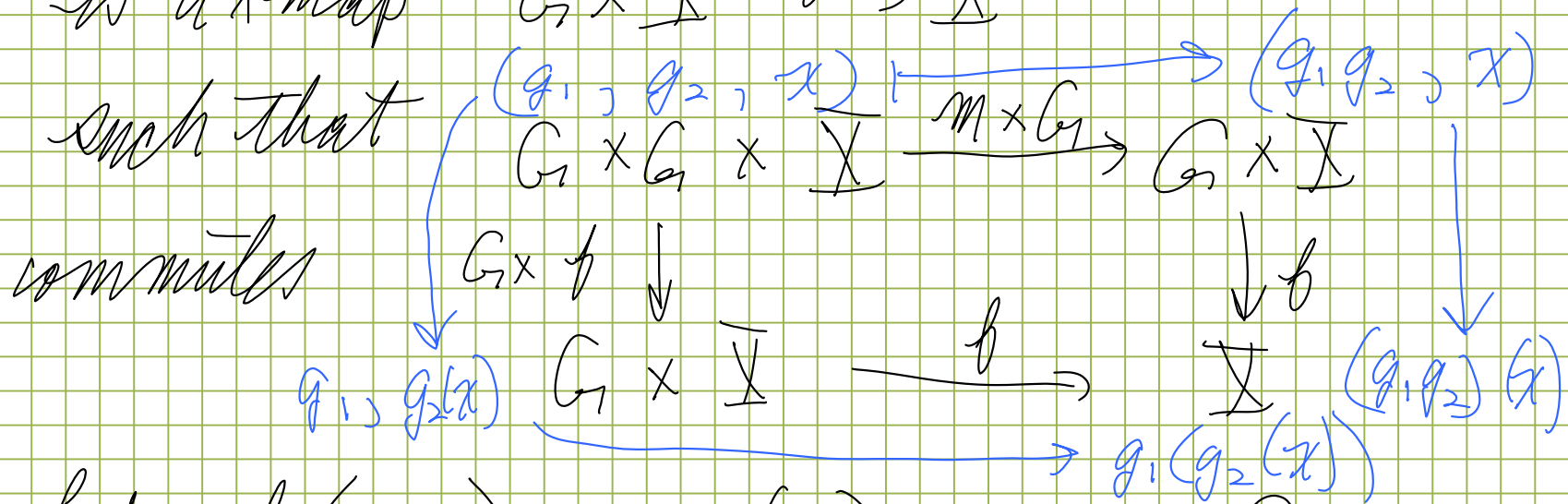


MATH 550

Note Title

8/31/2012

Definition Let G be a ^{topological} gp and X a top. space. A G -action on X is a ^{continuous} map $G \times X \rightarrow X$



Let $\phi(g, x) = : g(x)$
 $g \in G$
 $x \in X$

e.g. for the identity elt $e \in G$,
 $e(x) = x$.

Examples ① $G = \mathbb{Z} = \text{integers}$

$X = \mathbb{R} = \text{real line}$

$f(n, x) = n + x$, Action by translation

② $G = S_n = \text{symmetry gp on } n \text{ letters}$

$X = Y^n = n\text{-fold Cartesian product}$

G acts on X by permuting
co-ordinates.

$$X^G = \Delta(Y) = \{(y_1, y_1, \dots, y_1) \in Y^n\}$$

③ $G = S_n$, $X = \text{disjoint union of } n \text{ copies of } Y$

G permutes these n copies.

Definition: Given a G -space X
and a subgroup $H \subset G$, the fixed
point set of H ,

$$X^H = \{ x \in X : h(x) = x \text{ for all } h \in H \}$$

(It could be empty for $H \neq \{e\}$) For

$H \subset K \subset G$ then we have

$X^H \leftarrow X^K$ a restriction map.

Definition Given $x \in X$, its isotropy

$$\text{gp } G_x = \{ g \in G : g(x) = x \}$$

Definition Two points $x, x' \in X$
are in the same orbit if $\exists g \in G$
with $g(x) = x'$.

This is an equivalence relation, so
we get equivalence classes called
orbits. The set of orbits inherits
a topology from X . We get an
orbit space X/G .

Examples In (1) above, $X/G = S^1$
In (3) above $X/G = Y$

In $\textcircled{2}$ above $X/G = SP^n(Y)$

= n -fold symmetric product
of Y .

= space of unordered n -tuples

For $Y = S^2$, then $SP^n(S^2) \cong CP^n$.

EXERCISE.

Example 4

G = finite gp

X = finite set

(discrete topology)

If G acts on X , we say X is a
finite G -set.

Such things can be classified
The orbit of $x \in X$ has the form
 $G/G_x =$ left (right?) cosets of G_x

Example In (2) with Y finite

$$Y = \{a, b, c, d\} \quad n = 3$$

$$X = Y^3, \quad 64 \text{ elements.}$$

$$x = (a, a, b)$$

$$\text{orbit } Gx = \{(a, a, b), (a, b, a), (b, a, a)\}$$

In general each orbit is a finite

G -set has a conjugacy class of subgroups associated with it

Thm The set of isomorphism classes of finite G -sets is the free abelian monoid on the set of conjugacy classes of subgroups H of G .

In the above example

there are 4 orbits of the form G/H

"	12	"	G/S_2
	0		G/C_3

4

G/e