Recall Mackey functor chain $\mathcal{A}$ for $X = S^m$ \[ C_0 = \mathbb{C}_2 \]

For $m=2$, $m=3$:

\[
\begin{array}{cccc}
2 & 2 & 0 & 2 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

$\xi \mapsto [-1]$ \[ g = \text{sign rep} \]

Homology for $m=3$:

\[
\begin{array}{cccc}
2/2 & 0 & 2/2 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
2/2 & 0 & 2/2 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
2/2 & 0 & 2/2 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
2/2 & 0 & 2/2 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

$\xi(1-r) = \xi - \xi^2 = \xi - 1 = -(1-\xi)$ \[ \mathcal{Z} = \frac{2\xi}{1+\xi} \]

Def. A rep $\mathcal{C}$ is oriented if the matrix
associated to each elt in $G$ has det $1$.  Example: no is oriented iff $n$ is even

$$\det (-I) = (-1)^n$$

In a Mackey functor, $M$ for an abelian gp $G$, $M(G/H)$ has an action of the gp $G/H$.

**Remark:** Recall $\pi_i : x(G/H) = \prod_i (x^H)$

but in our examples ($n = 2, 3$)

$$H_x^i x(G/H) \neq H_x^i (xG)$$

because $x^G = S^n$ lent
\[ H^* X(\mathbb{Q}/\mathbb{Z}) \neq H^* S^0 \]

What is the meaning of this group? ??

Another example: \( G = C_4 \), \( X = S^R_{C_4} \)

\( R_4 \) = regular rep of \( C_4 \)

\( = \) permutation action on \( R^4 \).

This rep has a trivial 1-dimensional subspace generated by \( (1, 1, 1, 1) \)

\( R_4 \) = reduced regular rep

\( = \) subspace \( \{ (w, x, y, z) : w + x + y + z = 0 \} \)

\( S(R_4) \approx S^2 \). Let \( x \) be a generator of \( C_4 \).
\( \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \) is the eigenmatrix for \( Y \) in \( \mathbb{R}^4 \) in \( (1, -1, 1, -1) \).

\( Y \) reflects through equator (swapping the poles) and rotates around vertical axis by \( \pi / 2 \).

Cellular structure on \( Y = S(\mathbb{R}^4) \).

\( Y^0 = S^0 = 2 \) poles. We have 4 1-cells as shown.

And 4 2-cells.

A formulates the cells in each dim.
$S^4$ will have cells in dim $1, 2, 3$ that are double cones on the cells of $Y$, along with 2 0-cells that are fixed by $G$.

$S^4$ has a single 0-cell $2$ fixed 1-cell $3$ by $G$.

cells in dim $2, 3, 4$ obtained from the cells of $Y$ by joining with $S^1$.

The resulting reduced cellular chain complex is $N = C_2 \otimes C_4$. 
\[ 2 \cong \mathbb{Z}_G/4 \cong \mathbb{Z}_G \cong \mathbb{Z}_G \]

The homology of this complex is \( H_* \).

\( H_4 \) is generated by \( \alpha = (1-\gamma)(1+\gamma^2) = 1-\gamma+\gamma^2-\gamma^3 \) \( (1+\gamma)^2 = (1+\gamma)(1-\gamma)(1+\gamma^2) = (1-\gamma^2)(1+\gamma^2) = 1-\gamma^4 \)

\[ = 0 \]

The complex of Mackey functors is