Note Title 9/24/2012

tom Dieck Theorem 1979. For a finite gp G acting trivially on S^0, the G-spectrum $\Sigma^\infty S^0$ has

$$\underline{\pi}_0 \Sigma^\infty S^0 = \underline{A} = \text{Burnside ring Mackey functor}$$

where $\underline{A}(G/H) = A(H) =$ Burnside ring for H

Recall $A(G) =$ gp completion of the monoid of iso classes of finite G-sets

$= \mathbb{Z}\{G/H : H \text{ up to conjugacy}\}$

For $H \subset K \subset G$ then the restriction map

$$\begin{array}{ccc} \underline{A}(G/K) & \xrightarrow{\mathrm{Res}^K_H} & \underline{A}(G/H) \\ \| & & \| \\ A(K) & \xrightarrow{\text{forgetful}} & A(H) \end{array}$$

the transfer map is

$$\begin{array}{ccc} \underline{A}(G/K) & \xleftarrow{\mathrm{Tr}^K_H} & \underline{A}(G/H) \\ \| & & \| \\ A(K) & & A(H) \end{array}$$

$$K\text{-set} \ni K \times_H S \longleftrightarrow S \in H\text{-set}$$

$$\| $$

$$K \times S \,/\, (kh, s) \sim (k, hs) \qquad \begin{array}{l} h \in H \\ k \in K \\ s \in S \end{array}$$

$\mathrm{Res}^K_H \, \mathrm{Tr}^K_H$ is multiplication by $|K/H|$.

Note for G acting trivially on S^n,

$\underline{\pi}_n S^n = \underline{\mathbb{Z}}$, because $(S^n)^H = S^n$

tom Dieck's thm is an example of fixed points behaving badly, i.e. failing to commute with Σ^∞

$$(\Sigma^\infty S^n)^H \neq \Sigma^\infty (S^n)^H .$$

The slice filtration

Postnikov towers:

Given a space X, let $P^n X$ be the nth Postnikov section of X, a space with a map $X \longrightarrow P^n X$ s.t.

$$\pi_i P^n(X) = \begin{cases} \pi_i X & \text{for } i \le n \\ & \text{(iso induced by map)} \\ 0 & \text{for } i > n \end{cases}$$

This can be constructed as follows:

* choose a set of gens of $\pi_{n+1} X$
* for each one attach an $(n+2)$-cell

using the map $S^{n+1} \xrightarrow{f} X$

$$\begin{array}{c} X \\ \downarrow \\ X \cup_f e^{n+2} \end{array}$$

We get a map $X \to X'$ where

$\pi_{n+1} X' = 0$ and $\pi_i X' = \pi_i X$ for $i \le n$.

Do the same to $\pi_{n+2} X'$ and get X''

$$\pi_i X'' = \begin{cases} \pi_i X & \text{for } i \le n \\ 0 & \text{for } i = n+1, n+2. \end{cases}$$

In this way we get $P^n X$. It is independent of the choices made.

$$P_{n+1} X \longrightarrow X \xrightarrow{f_n} P^n X$$

$P_{n+1}X$ = fiber of f_n = n-connected cover of X

$$\pi_i\, P_{n+1} X = \begin{cases} 0 & \text{for } i \le n \\ \pi_i X & \text{for } i > n \end{cases}$$

$P_1 X$ = universal cover.

$$\begin{tikzcd}
 & & K(\pi_{n+1} X, n+1) =: P^{n+1}_{n+1} X \arrow[d] \\
P_{n+2} X \arrow[r] \arrow[d] & X \arrow[r, "f_{n+1}"] \arrow[d, equal] & P^{n+1} X \arrow[d, "g_n"] \\
P_{n+1} X \arrow[r] & X \arrow[r, "f_n"] & P^n X
\end{tikzcd}$$

$\boxed{\text{Postnikov tower}}$

n-connected cover $= P_{n+1} X$

$P^n X$ =: n th Postnikov section

$$X = \varprojlim P^n X$$

$$\text{point} = \varinjlim P^n X =$$

Let $\mathcal{S}$ be the category of spectra

$\mathcal{S}_{>n} \subset \mathcal{S}$ " n-connected "

P_{n+1} is a functor $\mathcal{S} \longrightarrow \mathcal{S}$ satisfying

1. For any X, $P_{n+1}X \in \mathcal{S}_{>n}$
2. For any $A \in \mathcal{S}_{>n}$ the map of function spectra $\mathcal{S}(A, P_{n+1}X) \longrightarrow \mathcal{S}(A, X)$ is a weak equivalence.

P^n is a functor $\mathcal{S} \longrightarrow \mathcal{S}$ satisfying

1. $P^n X$ is $\mathcal{S}_{>n}$-null, i.e. any map for $A \in \mathcal{S}_{>n}$ to $P^n X$ is null homotopic.

(2) For any $\mathcal{S}_{>n}$-null spectrum Z then the map

$$\mathcal{S}(P^n X, Z) \longrightarrow \mathcal{S}(X, Z)$$

is an weak equivalence.

Thm The subcategory $\mathcal{S}_{>n}$ of $\mathcal{S}$ determines the functors P^n and P_{n+1}.

The subcategory $\mathcal{S}_{>n}$ is that of spectra that can be built out of $\{S^m : m > n\}$ and is closed under cofibrations and colimits.