Note Title 9/26/2012

Recall $\mathcal{A}_{>n} = \mathcal{A}_{\geq n+1}$ = category of n-connected spectra

It leads (formally) to functors

P^n : nth Postnikov; kill π_i for $i > n$

P_{n+1} : n-connected cover.

We want to define subcategories $\mathcal{A}^G_{\geq n}$ in the G-equivariant case.

Will define it in terms of basic building blocks as before.

Let $H \subset G$ be a subgp

ρ_H = regular rep of $H = \mathbb{R}\{H\}$

$S^{m\rho_H}$ = H-space or spectrum

$\hat{S}(m,H) = G_+ \wedge_H S^{m\rho_H}$ = G-spectrum $m \in \mathbb{Z}$

G_+ = G with disjoint base pt.

The underlying space (for $m > 0$) is a wedge of $|G/H|$ copies of $S^{m|H|}$. G acts by permuting the wedge summands, each of which is H-invariant. H acts as indicated on $S^{m\rho_H}$

Let $\mathcal{A} = \left\{ \hat{S}(m, H), \Sigma^{-1}\hat{S}(m, H) : m \in \mathbb{Z} \text{ and } H \subset G \right\}$

These are called *slice cells*.

e.g. for $H = e$, $\hat{S}(m, e) = G_+ \wedge S^m$

$= \text{wedge of } |G| \text{ copies of } S^m$

$\hat{S}(m, e)^H = pt$ for $H \neq e$.

$$\Sigma^{-1}\hat{S}(m, e) = \hat{S}(m-1, e)$$

The dimension of a slice cell is that

of its underlying spheres.

Def $\mathcal{S}^G_{\geq n}$ is the subcategory of $\mathcal{S}^G$ (the category of G-spectra) "built" out of slice cells of dimension $\geq n$.

This subcategory of $\mathcal{S}^G$

1) Full (all morphisms included)

2) Closed under cofibers

$$X \xrightarrow{f} Y \xrightarrow{g} Z \quad \text{cofiber sequence}$$

If $X, Y \in \mathcal{S}^G_{\geq n}$ then $Z \in \mathcal{S}^G_{\geq n}$

3) Closed under extensions

If $X, Z \in \mathcal{S}^G_{\geq n}$ then $Y \in \mathcal{S}^G_{\geq n}$

4) Not closed under fibers

$Y, Z \in \mathcal{A}^{G}_{\geq n}$ does NOT IMPLY $X \in \mathcal{A}^{G}_{\geq n}$

5) Closed under infinite wedges and retracts.

6) Closed under direct limits.

$\mathcal{A}_{\geq n}$ has similar properties

This leads to functors P_n and P^{n-1} as before.

We will see later that if X is a slice of dim n, then

$P^n X = X \wedge H\underline{\mathbb{Z}}$

We have looked at this for $G = C_4$ and $n = 4$ $X = S^{\rho_4}$ and $G = C_2$ and $X = S^{k\rho_2}$, $n = 2k$

We found $\underline{\pi}_i P^n X = \underline{H}_i X$ can be nonzero for $0 < i < n$.

This differs from the classical case.

We have for a G-spectrum X

$$\begin{tikzcd}
 & \vdots \arrow[d] & & & \vdots \arrow[d] \\
 & P_{n+1}X \arrow[r] \arrow[d] & X \arrow[r] \arrow[d, equal] & P^n X \arrow[d] \\
 & P_n X \arrow[r] \arrow[d] & X \arrow[r] & P^{n-1} X \arrow[d] \\
 & \vdots & & \vdots
\end{tikzcd}$$

$$\mathcal{A}^G_{\geq n} \subset \mathcal{A}^G_{\geq n-1}$$

$\mathcal{S}^G$ = category of G-spectra.

This leads to the diagram

$$\cdots \longrightarrow P^n X \longrightarrow P^{n-1} X \longrightarrow \cdots$$

the slice tower for X.

$\varinjlim P^n X$ = point. and

$\varprojlim P^n X = X$

We also have slices

$$P^n_n X = \text{fiber of } P^n X \longrightarrow P^{n-1}$$

Classically this is $K(\pi_n X, n)$

In this case its π_* is not

concentrated in dimension n.
We get a spectral sequence with

$$E_1^{s,t} = \pi_{t-s} \, P_t^t X$$

Classically

$$E_1^{s,t} = \pi_{t-s} P_t^t X$$

$$= \begin{cases} \pi_t X & \text{if } s=0 \\ 0 & \text{if } s \neq 0 \end{cases}$$

This case is uninteresting.

Remarks about change of group.

Let $H \subset G$

$$\mathcal{S}^G \xrightarrow[\text{forgetful}]{i_H^*} \mathcal{S}^H \xrightarrow{G_+ \wedge_H -} \mathcal{S}^G$$

$$\mathcal{S}^G_{\geq n} \longmapsto \mathcal{S}^H_{\geq n} \longmapsto \mathcal{S}^G_{\geq n}$$

Def We say $X \geq n$ if $X \in \mathcal{S}^G_{\geq n}$
and $X \leq n$ if the map $X \longrightarrow P^n X$
is an equivalence.

Classically $X \geq n$ means X is $(n-1)$-connected
$\pi_i X = 0$ unless $i \geq n$.
$X \leq n$ if $\pi_i X = 0$ unless $i \leq n$.

In the Legvn case

$X \geq n$ means $[W, X]^{G} = 0$

when W is a slice cell of dim $< n_0$

$X \leq n$ means $[W, X]^{G} = 0$

when W is a slice cell of dim $> n_0$