Def: A $G$-spectrum $X$ is in $\Sigma^n G$ if it is an object in $\mathcal{A} _{> n}$.

Prop 2.8 (from Hill's Primer)

For all $H \unlhd G$ and all $n \geq 1$,

$C_{n+1} H \times S^n \simeq n^0$.

Proof for $G_1 = G_2$: Obvious for $H = e$.

$C_{n+1} H \times S^n \simeq S^n$. Let $\sigma$ denote the sign rep of $G_1$. Let $P_0 = 1 + s \in R_0(G)$.

We have a fiber sequence $0 \to k \to n$

$C_{n+1} S^{2n-k-1} \to S^{n+(n-k)/2} \to S^{n+(n-k)/2} \to G_{n+1} S^{2n-k} \to S^{2n-k-1} \geq 2n-k$.
e.g. \[ S^1 \rightarrow S^{n+\delta} \rightarrow G^{n+1} \rightarrow S^2 \]

\[ n=1 \]
\[ k=0 \]

\[ \text{reflection} \quad S^2 \cup S^2 \]

Show by induction on \( k \):
\[ S^{n+(n-k)+\delta} \geq 2n-k \]

For \( k=0 \), we have \( S^{n+n\delta} = S^{n\delta} \geq (2n)^{-\delta} \).

Use the cofiber sequence above.
For \( k=n \), we have \( S^n = n \).

Note \( S^1 = S^1 (S^0)^x \), QED

Example: \( G = C_4 \), consider skeleton of \( S^4 \)

\[ p_4 = 1 + 6 + \chi \text{ where } \chi = \text{sign rep} \]

\[ x \rightarrow [-1] \]
and \( x \) is rotated in \( \mathbb{R}^2 \) by \( \pi/2 \)

\[
Y \mapsto \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}
\]

\[
S^1 \rightarrow S^{1x6} \rightarrow 3 \text{- skeleton} \rightarrow S^{p_4}
\]

1-skeleton \quad 2-skeleton

\[
G_0 \cong S^1 \quad G_1 \cong S^2 \quad G_2 \cong S^3 \quad G_3 \cong S^4
\]

Can show each \( k \)-skeleton is \( \geq k \).

\( G_2 \) for all \( G \)-spectra \( X \) and \( n \geq -1 \), \( p^{n-1}X \) is \( n \)-coconnected, i.e., \( \Pi_i p^{n-1}X = 0 \) for \( i \geq n \).

\( \text{Pf: By construction, for } W \geq n \)

\[
[EW, p^{n-1}X]_0 = 0 \quad s_i \geq i \quad \text{for } i \geq 1.
\]

\[
\implies \Pi_i (p^{n-1}X) = 0 \quad \text{for } i \geq n
\]
More consequences

1. $A_{> 0} = \text{category of} (-1)-\text{connected spectra}$

2. $A_{> 1} = \text{category of} (-2)-\ldots$

Remark: For any Mackey functor $M$
there is an Eilenberg-MacLane spectrum $HM$ with

\[
\pi_i H M = \begin{cases} M & \text{for } i = 0 \\ 0 & \text{for } i \neq 0. \end{cases}
\]

Corollary 2.12: For any $G$-spectrum $X$

\[
P^n X = \Sigma^{-1} H \pi_1 X
\]

Remark: Classically

\[
P^n X = \Sigma^n H \pi_0 X
\]
Not true for $n \neq -1$ in any case.

\[
\ell_{\ell_{n-1}} X = \sum P_{n} H \ell_{\ell_{n+1}} X
\]

\[
\not= \sum P_{n} H \ell_{\ell_{n-1}} X
\]

Denote the $n$th "$\ell_{n}$-slice" of $X$ by $G_{n} P_{n} X$. Then for any $H \in G_{n}$

\[
H P_{n} X = i_{n} (G_{n} P_{n} X)
\]

for $n \geq -1$.

It is true for $H = e_{n}$.