Note Title 9/28/2012

Def A G-spectrum X is $\geq n$ if it is an object in $\mathcal{S}^{G}_{\geq n}$.

Prop 2.8 (from Hill's primer)

For all $H \subset G$ and all $n \geq -1$,

$$G_+ \wedge_H S^n \geq n_0$$

Proof for $G = C_2$: Obvious for $H = e$.

$G_+ \wedge_H S^n = S^n_0$. Let σ denote the sign rep of G, so $\rho_G = 1 + \sigma \in RO(G)$

We have a cofiber sequence $0 \leq k < n$

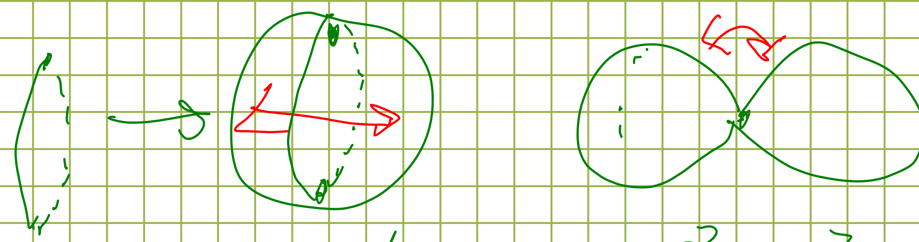
$$\boxed{G_+ \wedge S^{2n-k-1} \longrightarrow S^{n+(n-k-1)\sigma} \longrightarrow S^{n+(n-k)\sigma}} \longrightarrow G_+ \wedge S^{2n-k}$$

$\geq 2n-k-1$ $\qquad$ $\geq 2n-k-1$ $\qquad$ $\geq 2n-k$

e.g. $S^1 \longrightarrow S^{1+\sigma} \longrightarrow G_+ \wedge S^2$

$n=1$

$k=0$



reflection

$S^2 \vee S^2$

Show by induction on k,

$$S^{n+(n-k)\sigma} \geq 2n-k$$

For $k=0$ we have $S^{n+n\sigma} = S^{n\rho} = (2n)$-slice cell

Use the cofiber sequence above.

For $k=n$ we have $S^n \geq n$.

Note $S^{-1} = \Sigma^{-1}(S^{0\rho_G})$. QED

Example $G = C_4$. Consider skeleton of S^{ρ_4}

$\rho_4 = 1+\sigma+\lambda$ where σ = sign rep

$\gamma \mapsto [-1]$

and γ is rotation in $\mathbb{R}^2$ by $\pi/2$

$$\gamma \longmapsto \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{tikzcd}
S^1 \arrow[r] & S^{1+\sigma} \arrow[r] \arrow[d] & \text{3-skeleton} \arrow[r] \arrow[d] & S^{\rho_4} \arrow[d] \\
& G_+ \wedge_{C_2} S^2 & G_+ \wedge S^3 & G_+ \wedge S^4
\end{tikzcd}$$

1-skeleton $= S^1$, $S^{1+\sigma} =$ 2-skeleton

Can show each k-skeleton is $\geq k$.

<u>Cor 2.9</u> For all G-spectra X and $n \geq -1$, $P^{n-1}X$ is n-coconnected, i.e. $\pi_i P^{n-1} X = 0$ for $i \geq n$.

<u>Pf</u> By construction, for $W \geq n$

$$[W, P^{n-1}X]^G = 0 \qquad S^i \geq i \text{ for } i \geq -1.$$

so $\pi_i(P^{n-1}X) = 0$ for $i \geq n$

More consequences

① $\mathcal{S}_{\geq 0}$ = category of (-1)-connected G-spectra

② $\mathcal{S}_{\geq -1}$ = category of (-2)- ...

<u>Remark</u> For any Mackey functor $\underline{M}$ there is an Eilenberg-MacLane spectrum $H\underline{M}$ with

$$\underline{\pi}_i H\underline{M} = \begin{cases} \underline{M} & \text{for } i = 0 \\ 0 & \text{for } i \neq 0. \end{cases}$$

<u>Cor 2.12</u> For any G-spectrum X,

$$P_{-1}^{-1} X = \Sigma^{-1} H \underline{\pi}_{-1} X$$

<u>Remark</u> Classically $P_n^n X = \Sigma^n H \pi_n X$

Not true for $n \neq -1$ in equiv case.

$$P^{|G|-1}_{|G|-1} X = \sum^{P_G} {}^{H}\Pi_{|G|-1} X$$

$$\neq \sum^{|G|} {}^{H}\Pi_{|G|-1} X$$

Denote the nth "G-slice" of X by ${}^{G}P^n_n X$. Then for any $H \subset G$

$${}^{H}P^n_n X = i_H^* ({}^{G}P^n_n X) \quad \text{for } n \geq -1.$$

It is true for $H = e$.