MU(\mathbb{C}_8) = N_{\mathbb{C}_2}^C \overset{\text{MU}}{\rightarrow} \mu = MU(4)

D \equiv \mathbb{T}_x^* \text{MU}(\mathbb{C}_8)

\Xi = 52 = \mathbb{T}_x^* \text{MU}(\mathbb{C}_8)

\Xi 0 = 52 = 52 C_8 = 52 e_8

Periodicity \quad \mathbb{T}_{256} \ast \Xi = \mathbb{T}_x \Xi

Will look at slice SS from MU(\mathbb{C}_8)

We need \text{G} = C_2 \rightarrow C_4 \rightarrow C_8

Will need RO(\text{G}) - graded slice SS
Let $V$ be a virtual rep of $G$

\[ E_{2k}^V = \prod_{V + k - s} \mathbb{P}^V \quad \text{for} \quad X = \prod_{V + k - s} X \]

We need the case $V = -6$, $\delta = \text{sign rep}$

We will say twist $= -1$.

Special elements

1. For any $\mu \in V$ we have $S^\mu \mapsto S^\mu \in \prod_{V - s} G \leq S^0$

\[ a_{\mu \nu} = a_\mu a_\nu \quad a = a_\delta \leq E_{2V}^{-1, -\delta} \]

(Stationary image)

For any $G$-spectrum $X$

\[ \pi^* \otimes E_{2V} X = \alpha^* \pi^* X \quad \text{where} \quad \pi^* \otimes E_{2V} X \]
2. \( M_\mu \in \Pi_{i \in G_\nu} M_\nu (C_\alpha) \)
\[ \overline{M}_\mu \in \Pi_{i \in G} M_\nu \]
\[ \Pi_{i \in G} M_\nu \]
\[ \overline{N}_{C_\alpha} \overline{M}_\mu \in \Pi_{i \in G} M_\nu \]
\[ f^{-1} = C_{i \in G} N(\bar{m}) \]
\[ \in \mathbb{E} \]
\[ \bar{p}_\alpha = \text{regular rep of } C_\alpha \]
\[ \bar{p}_\alpha = \text{regular rep of } C_\alpha \]
\[ \bar{p}_\alpha = \text{reduced regular rep of } C_\alpha \]
\[ q = 1671 \]

3. For an oriented \( V \) we have
\[ H_{G_\nu} \left( \psi V \right) \rightarrow H_{G_\nu} \left( \psi V \right) \rightarrow \text{orientation} \]
\[ M_\nu \in \Pi_{i \in G} M_\nu \]
For any $V$, $2V$ is oriented so $M_{2V}$ exists.

$U_{V \otimes W} = U_V U_W$

$M = M_{20} \in E^{0,2-26}_2$

Remark: above can be constructed geometrically.

$C_3$-mfd $M$ with $V_M = V_1 \otimes V_2 \otimes V_3 \otimes V_4$

where $V_i$ is complex and

$\gamma: V_1 \rightarrow V_i$ and $\gamma^* : V_i \rightarrow$ anti-linear involution

even spectrum $\rightarrow MU(C(8))$ on trivial universe
In the slice $SS$ for $V = -l$, treat $-l$.

$F_2^2 \left[ a, f_1, u \right] \sqrt{2a, 2f_1} \to E_{(a,t-1)}$

**Thm:** This is an iso for $s = (q-1)(t-s-1)$

For $l = 2^k$

- killed by lowerdiffs
- other stuff
- $f_i$ lies on line of slope $(q-1)$
- $s = (q-1)(t-s-2^k)$

$a^2 u^{k-1}$
Three Differentials Thm. In the slice $SS$ for $MU^{(n)}$ the shift on $H_{k-1}$ is

$$d_{1+(2^k-1)}(H_{k-1}) = A^{2^k} b_{2^k-1}$$

Proof:

$$\partial_g MU^{(G)} = MO$$ and

$$\pi_* MO = 2/2^G h_i : i \neq 2^k-1$$

$$f_i \mapsto h_i$$

$$f_{2^k-1} \mapsto 0$$

Argue by induction on $k$. Assume true for $k' < k$, so lower powers of $H$ are done.
Inverting a gives SS converging to $\mathcal{M}_d \left[ a^{+1} \right]$. A diff cannot hit $f_i$ for $i \neq 2^k - 1$, the only possible diff on $n^{2^k-1}$ is the one indicated, i.e.

Also $f_{2^k}$ must be killed by a power of $q$. This differential is the only thing that can kill it. QED
Inverting elements
\[ \Delta^G_k = N^G_{C_2}(\overline{M}_{2k-1}) \in \prod_{G, \mathfrak{P} \leq 1} \text{MU}^{\text{cyl}}(G) \]

\[ \bar{\Delta}^G_k \]

Inverting \( \bar{\Delta}^G_k \) makes \( U^{k-1} \) a form cycle.

For \( \text{MU}^{\text{cyl}}(G) \), \( d(U^{k-1}) = a_z^{k+1} f_{2k-1}^{k+1} \). We will show this gets killed earlier.

\[ f_{2k-1}^{k+1} \bar{\Delta}^G_k = (a_z^{k+1})^N(\overline{M}_{2k-1}) \]
\[ = (a_z^{k+1})^N(\overline{M}_{2k-1}) (a_z^{k+1})^N(\overline{M}_{2k-1}) \]
\[ = a_z^{k+1} \bar{\Delta}^G_{k+1} f_{2k-1}^{k+1} \]
\[ a^2 \left[ \Delta_k (g) \right] (x) = \frac{1}{2} \Delta_{k+1} (g) \quad \text{for } n = 0, \ldots, n_k - 1 \]

etc.

QED

**Norm function**

If \( H \subseteq G \) and \( V \) is \( H \)-rep.

then \( N_{g_H^c}^H V = \Lambda_{g_H^c}^H (V) \)

There is an identity

\[ U \Lambda_{g_H^c}^H V = N_{g_H^c}^H (UV) = \Lambda_{g_H^c}^H (UV) \]
From this we find
\[ M_{2P_8} = M_{86_8} N_{C_4} (M_{46_4}) N_{C_2} (M_{26_2}) \]

inverting \( \overline{(2)} \) makes \( M_{326_2} = M_{16_2} \) a form cycle

\( \overline{(4)} \)
\( \overline{(2)} \)
\( \overline{(1)} \)

Then \( D = \overline{(8)} N_{C_8} (\Delta (4)) N_{C_4} (\Delta (2)) \) makes \( M_{326_8} p_8 \) a form cycle.
Define \( \Delta_1^{(b)} = M_{2p_b} (\Delta_1^{(b)})^2 \)

\[ (\Delta_1^{(b)})^2 = M_{32p_b} (\Delta_1^{(b)})^2 \in \pi_{256-b} D^{-1} MU((b)) \]

If we forget to underlying spectrum

\[ M_{32p_b} : S^{256-32p_b} \longrightarrow D^{-1} MU((b)) \]

becomes a unit \( \Delta_0 \)

\( (\Delta_1^{(b)})^1_b \) is a form cycle forgetting to unit

\[ \pi_x D^{-1} MU((b)) (\Delta_1^{(b)})^1_b \xrightarrow{\Delta_0} \pi_x D^{-1} MU((b)) \]

This means it also gives an equivalence
of its fixed fits

$D^{-1}MU((c))^{-1}h^S_t$ is $256$-periodic.